“Turbo Codes for xDSL modems”

Juan Alberto Torres, Ph. D.
VOCAL Technologies, Ltd. (http://www.vocal.com)
200 John James Audubon Parkway
Buffalo, NY 14228, USA
Phone: +1 716 688 4675
Fax: +1 716 639 0713
Email: jatorres@vocal.com
Current FEC in ADSL modems

Reed-Solomon (1)

- Actually G.992.1 & G.992.2 uses mandatory Reed-Solomon for FEC to avoid Impulse Noise.

- Reed-Solomon ensured Error Free operation for any impulse noise < than 2 DMT symbols latency < 10 ms and data rate < 6.4 Mbps with a redundancy around 12 %.

- $R$ redundant check bytes $c_0, c_1, ... , c_{R-2}, c_{R-1}$ are appended to $K$ message bytes $m_0, m_1, ... , m_{K-2}, m_{K-1}$ to form a Reed-Solomon codeword of size $N = K + R$ bytes. The check bytes are computed from the message byte using the equation:

$$C(D) = M(D)D^R \text{ modulo } G(D)$$

$$M(D) = m_0D^{K-1} + m_1D^{K-2} + ... + m_{K-2}D + m_{K-1}$$

$$C(D) = c_0D^{R-1} + c_1D^{R-2} + ... + c_{R-2}D + c_{R-1}$$

$$G(D) = \prod_{i=0}^{R-1}(D + \alpha^i)$$
# Current FEC in ADSL modems

Reed-Solomon (2)

Table 7-7/G.992.1 – Minimum FEC coding capabilities for ATU-C

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fast buffer</th>
<th>Interleaved buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parity bytes per R-S codeword</td>
<td>( R_F = 0, 2, 4, 6, 8, 10, 12, 14, 16 ) (Note 1)</td>
<td>( R_I = 0, 2, 4, 6, 8, 10, 12, 14, 16 ) (Notes 1 and 2)</td>
</tr>
<tr>
<td>DMT symbols per R-S codeword</td>
<td>( S = 1 )</td>
<td>( S = 1, 2, 4, 8, 16 )</td>
</tr>
<tr>
<td>Interleave depth</td>
<td>Not applicable</td>
<td>( D = 1, 2, 4, 8, 16, 32, 64 )</td>
</tr>
</tbody>
</table>

**NOTE 1** – \( R_F \) can be > 0 only if \( K_F > 0 \), and \( R_I \) can be > 0 only if \( K_I > 0 \).

**NOTE 2** – \( R_I \) shall be an integer multiple of \( S \).

Table 8-3/G.992.1 – Minimum FEC coding capabilities for ATU-R

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fast buffer</th>
<th>Interleaved buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parity bytes per R-S codeword</td>
<td>( R_F = 0, 2, 4, 6, 8, 10, 12, 14, 16 ) (Note 1)</td>
<td>( R_I = 0, 2, 4, 6, 8, 10, 12, 14, 16 ) (Notes 1 and 2)</td>
</tr>
<tr>
<td>DMT symbols per R-S codeword</td>
<td>( S = 1 )</td>
<td>( S = 1, 2, 4, 8, 16 )</td>
</tr>
<tr>
<td>Interleave depth</td>
<td>Not applicable</td>
<td>( D = 1, 2, 4, 8 )</td>
</tr>
</tbody>
</table>

**NOTE 1** – \( R_F \) can be > 0 only if \( K_F > 0 \) and \( R_I \) can be > 0 only if \( K_I > 0 \).

**NOTE 2** – \( R_I \) shall be an integer multiple of \( S \).
Current FEC in ADSL modems

Trellis Code Modulation

- Optional in G.992.1 as inner encoder.

\[ u_{z'} \rightarrow w_{y-1} \]
\[ u_{z'-1} \rightarrow w_{y-2} \]
\[ \vdots \]
\[ u_{z'-y+3} \rightarrow w_2 \]
\[ u_{z'-y+2} \rightarrow v_{z'-y} \]
\[ u_{z'-y+1} \rightarrow v_{z'-y-1} \]
\[ u_4 \rightarrow v_2 \]
\[ u_3 \rightarrow u_2 \rightarrow u_1 \rightarrow u_0 \rightarrow v_1 = u_1 \oplus u_3 \]
\[ v_0 = u_3 \]
\[ w_1 = u_0 \oplus u_1 \]
\[ w_0 = u_2 \oplus u_3 \]
Next FEC in ADSL modems

- It has been agreed that the committee “shall develop a proposal for improved coding gain”.

- It is agreed that proposals for coding techniques shall present:
  - Net coding Gain and Latency for $10^{-3}$, $10^{-7}$ and $10^{-9}$.
  - Results with and without R-S as outer encoder.
  - Results for spectral efficiencies of 4 bit/s/Hz and 12 bit/s/Hz.
  - Complexity (also relative to G.992.1).
  - Impulse and AWGN noise results and error statistics.
  - Impulse noise defined as 2 DMT symbols with 5, 10 and 15 dB more noise than the Reference Level (RL).
  - Reference level is level for $10^{-7}$ uncoded.
  - Code rate and BER.
  - Estimation of the expected Turbo Code error floor.
Next FEC in ADSL modems

Turbo Codes (1)

- Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel.

- Turbo codes were first presented in the ICC 93 by C. Berrou, A. Glavieux and P. Thitimajshima.

- Turbo codes had been proposed for ADSL modems since August 1998 by VOCAL Technologies Ltd (Q4/SG15 AB-093).

- Turbo codes use at least two Parallel Concatenated Convolutional Codes that use systematic recursive codes (SRC) and iterative decoding techniques with soft-in-soft-out (SISO) decoders.

- These codes provide from the same information bit \( d_i \), two parity bits \( p_i \) and \( q_i \).
Next FEC in ADSL modems

Turbo Codes (2)

- Between the two SRC, there is an interleaver that change the order of the information bits before they enter the second SRC.

- The design of the interleaver is very important.

- It is possible to puncture (select alternately) the parity bits to reduce the redundancy with a small degradation in the performance.
What is next in xDSL?

Turbo codes for xDSL modems

**Next FEC in ADSL modems**

Turbo Codes (3)

- Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:
  - Inmarsat’s new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
  - The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification.
  - NASA’s next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
  - The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
  - The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.
Next FEC in ADSL modems

Turbo Codes (4)

- Equations:

\[ \frac{E_b}{N_0}[dB] = \frac{E_s}{N_0}[dB] - 10 \log_{10}(\eta) [dB] \]

\[ \frac{C}{N_0} [dB-Hz] = \frac{E_b}{N_0} [dB] + 10 \log_{10}(R_b) [dB-Hz] \]

\[ SNR = \frac{E[|a_k|^2]}{E[|w_k|^2]} = \frac{E[|a_k|^2]}{D\sigma_N^2} = \frac{E_{av}}{D\sigma_N^2} \]

\[ SNR = \frac{E_s}{D\frac{N_0}{2}} = \frac{\eta E_b}{D\frac{N_0}{2}} \]

\[ \sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-l} \]
Next FEC in ADSL modems

Turbo Codes (5)

• Turbo code proposed by VOCAL Technologies Ltd. (BA-020 and HC-073)

• Use of square constellations that allow for use of very efficient blind equalization techniques, effectively maintaining I and Q independent.

• Gray mapping with the information bits are more protected than parity bits. This is specially effective for impulse noise, because the parity does not provide information if an impulse noise is present.

• The Soft-Output Turbo decoder can provide information to the Reed-Solomon outer decoder to increase its performance by 2. This means that with the Soft-Output turbo decoder R-S can correct up to R symbols.
**Next FEC in ADSL modems**

Turbo Codes (6)

- Coding and modulation for 4 Bit/s/Hz spectral efficiency.

Puncturing and Mapping for Rate 4/6 64 QAM

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>-</td>
<td>q₃</td>
<td>-</td>
</tr>
<tr>
<td>8AM symbol (I)</td>
<td></td>
<td></td>
<td>(d₁,d₂,p₁)</td>
<td></td>
</tr>
<tr>
<td>8AM symbol (Q)</td>
<td></td>
<td></td>
<td>(d₃,d₄,q₃)</td>
<td></td>
</tr>
<tr>
<td>64 QAM symbol (I,Q)</td>
<td></td>
<td></td>
<td>(I,Q)=(d₁,d₂,p₁,d₃,d₄,q₃)</td>
<td></td>
</tr>
</tbody>
</table>
Next FEC in ADSL modems

Turbo Codes (7)

• Coding and modulation for 4 Bit/s/Hz spectral efficiency.

\[ E_{av} = \frac{1+9+25+49}{4} A^2 = 21 A^2 \]

\[ \sigma_N^2 = E_{av} \left( \frac{2 \eta E_b}{N_0} \right)^{-1} = 21 A^2 \left( \frac{2 x 2 x E_b}{N_0} \right)^{-1} = 5.25 A^2 \left( \frac{E_b}{N_0} \right)^{-1} \]

• Bit Probabilities: The 8 AM symbol is defined as \( u^k = (u_1^k, u_2^k, u_3^k) \), where \( u_1^k \) is the most significant bit and \( u_3^k \) is the least significant bit. The following set can be defined.

  bit-1-is-1 = \{ A_4, A_5, A_6, A_7 \}
  bit-2-is-1 = \{ A_0, A_1, A_6, A_7 \}
  bit-3-is-1 = \{ A_1, A_2, A_5, A_6 \}

\[
LLR(u^k_n) = \log \left( \frac{\sum_{A_i \in \text{bit-1-is-1}} \exp \left( -\frac{1}{2 \sigma_N^2} \left\| R^k - A_i \right\| \right)}{\sum_{A_j \in \text{bit-2-is-0}} \exp \left( -\frac{1}{2 \sigma_N^2} \left\| R^k - A_j \right\| \right)} \right)
\]
What is next in xDSL?

Next FEC in ADSL modems

Turbo Codes (8)

- For an interleaver size of 10,400 information bits using the interleaver defined in the 3GPP (3G TS 25.212 v3.2.0) recommendation.

BER for Rate 4/6 64QAM N=10400 bits AWGN Channel
Next FEC in ADSL modems

Turbo Codes (9)

- Very resistance to impulse noise because of the probability of no error:
What is next in xDSL?

Next FEC in ADSL modems

Turbo Codes (10)

- Coding and Modulation for 12 Bit/s/Hz spectral efficiency.

Puncturing and Mapping for Rate 4/6 64 QAM

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
<th>d₅</th>
<th>D₆</th>
<th>d₇</th>
<th>d₈</th>
<th>d₉</th>
<th>d₁₀</th>
<th>d₁₁</th>
<th>d₁₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>q₇</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>128AM symbol (I)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(d₁, d₂, d₃, d₄, d₅, d₆, p₁)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>128AM symbol (Q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(d₇, d₈, d₉, d₁₀, d₁₁, d₁₂, q₇)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16384 QAM symbol (I, Q)</td>
<td>(d₁, d₂, d₃, d₄, d₅, d₆, p₁, d₇, d₈, d₉, d₁₀, d₁₁, d₁₂, q₇)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Next FEC in ADSL modems

Turbo Codes (11)

- Coding and modulation for 12 Bit/s/Hz spectral efficiency.

\[
E_{av} = 5,461 A^2
\]

\[
\sigma_N^2 = E_{av} \left( \frac{2N E_b}{N_0} \right)^{-1} = 5,461 A^2 \left( \frac{2 \times 6 x E_b}{N_0} \right)^{-1} = 455.08 A^2 \left( \frac{E_b}{N_0} \right)^{-1}
\]

- Bit Probabilities: The 128 AM symbol is defined as \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k, u_9^k, u_{10}^k, u_{11}^k, u_{12}^k, u_{13}^k, u_{14}^k) \) where \( u_1^k \) is the most significant bit and \( u_{14}^k \) is the least significant bit.
Next FEC in ADSL modems

Turbo Codes (12)

- For an interleaver size of 31,200 information bits using the interleaver defined in the 3GPP (3G TS 25.212 v3.2.0) recommendation.
What is next in xDSL?

### Next FEC in ADSL modems

**Turbo Codes (13)**

- Simulation Results: Net Coding Gain without Reed-Solomon

<table>
<thead>
<tr>
<th>Bit/Tone</th>
<th>Tones</th>
<th>Interleaver Size</th>
<th># of DMT symbols</th>
<th>Latency (Tx+Rx) ms &lt; 10⁻³</th>
<th>10⁻⁷</th>
<th>10⁻⁹ extrap.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>5,200</td>
<td>13</td>
<td>10.0</td>
<td>4.60</td>
<td>7.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>2</td>
<td>1.5</td>
<td>3.70</td>
<td>4.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>1</td>
<td>0.7</td>
<td>3.30</td>
<td>3.62</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>10,400</td>
<td>13</td>
<td>10.0</td>
<td>4.60</td>
<td>7.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,600</td>
<td>2</td>
<td>1.5</td>
<td>4.10</td>
<td>6.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>1</td>
<td>0.7</td>
<td>3.70</td>
<td>4.92</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>15,600</td>
<td>13</td>
<td>10.0</td>
<td>4.10</td>
<td>5.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>2</td>
<td>1.5</td>
<td>3.60</td>
<td>5.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,200</td>
<td>1</td>
<td>0.7</td>
<td>3.00</td>
<td>3.91</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>31,200</td>
<td>13</td>
<td>10.0</td>
<td>4.10</td>
<td>6.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4,800</td>
<td>2</td>
<td>1.5</td>
<td>3.60</td>
<td>5.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>1</td>
<td>0.7</td>
<td>3.60</td>
<td>5.51</td>
</tr>
</tbody>
</table>
What is next in xDSL?

Next FEC in ADSL modems

Turbo Codes (14)

- Errors due to impulse noise without Reed-Solomon: The impulse noise is defined as 2 consecutive DMT symbols with an increase AWGN respect to the reference noise level of a carrier-to-noise ratio of the uncoded system.

<table>
<thead>
<tr>
<th>Bits/Tone</th>
<th># Tones</th>
<th>Interleaver Size</th>
<th>RL + 2.5 dB</th>
<th>RL + 5 dB</th>
<th>RL + 7.5 dB</th>
<th>RL + 10 dB</th>
<th>RL + 12.5 dB</th>
<th>RL + 15 dB</th>
<th>RL + 17.5 dB</th>
<th>RL + 20 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td></td>
<td>5,200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
<td>0</td>
<td>0</td>
<td>39</td>
<td>65</td>
<td>104</td>
<td>140</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>50</td>
<td>89</td>
<td>127</td>
<td>161</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
<td>10,400</td>
<td>0</td>
<td>0</td>
<td>400</td>
<td>116</td>
<td>187</td>
<td>252</td>
<td>346</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,600</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>127</td>
<td>189</td>
<td>267</td>
<td>363</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>116</td>
<td>187</td>
<td>252</td>
<td>346</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td></td>
<td>15,600</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>58</td>
<td>130</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,400</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>78</td>
<td>121</td>
<td>171</td>
<td>216</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,200</td>
<td>0</td>
<td>0</td>
<td>43</td>
<td>98</td>
<td>129</td>
<td>188</td>
<td>255</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
<td>31,200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>175</td>
<td>313</td>
<td>482</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4,800</td>
<td>0</td>
<td>0</td>
<td>75</td>
<td>177</td>
<td>254</td>
<td>341</td>
<td>462</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,400</td>
<td>0</td>
<td>0</td>
<td>80</td>
<td>166</td>
<td>244</td>
<td>345</td>
<td>457</td>
</tr>
</tbody>
</table>
**Next FEC in ADSL modems**

Turbo Codes (15)

- It is interesting that for the large turbo decoders the impulse errors still tend to stay within the 2 DMT symbols. This implies a moderately large turbo coder of 5 ms follow by a convolutional interleaver/Reed-Solomon of 10 ms should create both robust performance and good impulse resistance.
**Next FEC in ADSL modems**

**Turbo Codes (16)**

- Simulation Results: Coding Gain with Reed-Solomon

<table>
<thead>
<tr>
<th>Bit/Tone</th>
<th>Tones</th>
<th>Interleaver Size</th>
<th># of DMT symbols</th>
<th>Latency (Tx+Rx) ms (&lt;)</th>
<th>10(^{-3})</th>
<th>10(^{-7})</th>
<th>10(^{-9}) extrapol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td></td>
<td>5,200</td>
<td>13</td>
<td>10.0</td>
<td>5.00</td>
<td>8.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
<td>2</td>
<td>1.5</td>
<td>3.50</td>
<td>7.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td>1</td>
<td>0.7</td>
<td>3.50</td>
<td>6.42</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td>10,400</td>
<td>13</td>
<td>10.0</td>
<td>5.30</td>
<td>8.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,600</td>
<td>2</td>
<td>1.5</td>
<td>4.60</td>
<td>7.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
<td>1</td>
<td>0.7</td>
<td>3.50</td>
<td>7.12</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td></td>
<td>15,600</td>
<td>13</td>
<td>10.0</td>
<td>4.40</td>
<td>7.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,400</td>
<td>2</td>
<td>1.5</td>
<td>4.60</td>
<td>7.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1,200</td>
<td>1</td>
<td>0.7</td>
<td>4.10</td>
<td>6.81</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td>31,200</td>
<td>13</td>
<td>10.0</td>
<td>4.40</td>
<td>7.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4,800</td>
<td>2</td>
<td>1.5</td>
<td>4.40</td>
<td>7.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,400</td>
<td>1</td>
<td>0.7</td>
<td>4.60</td>
<td>7.41</td>
</tr>
</tbody>
</table>
**Next FEC in ADSL modems**

**Turbo Codes (17)**

- Simulation Results: NET Coding Gain with Reed-Solomon

<table>
<thead>
<tr>
<th>Bit/Tone</th>
<th>Tones</th>
<th>Interleaver Size</th>
<th># of DMT symbols</th>
<th>Latency (Tx+Rx) ms &lt; 10⁻³</th>
<th>10⁻⁷</th>
<th>10⁻⁹ extrap.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>5,200</td>
<td>13</td>
<td>3.42</td>
<td>7.04</td>
<td>8.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>2</td>
<td>1.78</td>
<td>5.40</td>
<td>6.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>1</td>
<td>1.94</td>
<td>4.86</td>
<td>5.88</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>10,400</td>
<td>13</td>
<td>3.72</td>
<td>7.24</td>
<td>8.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,600</td>
<td>2</td>
<td>2.88</td>
<td>6.00</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>1</td>
<td>1.94</td>
<td>5.56</td>
<td>6.88</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>15,600</td>
<td>13</td>
<td>0.20</td>
<td>3.51</td>
<td>4.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>2</td>
<td>0.02</td>
<td>2.83</td>
<td>3.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,200</td>
<td>1</td>
<td>-0.24</td>
<td>2.47</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>31,200</td>
<td>13</td>
<td>1.06</td>
<td>4.37</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4,800</td>
<td>2</td>
<td>0.76</td>
<td>3.57</td>
<td>4.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>1</td>
<td>0.26</td>
<td>3.07</td>
<td>3.99</td>
</tr>
</tbody>
</table>
**Next FEC in ADSL modems**

Turbo Codes (18)

- Errors due to Impulse noise with Reed-Solomon.

<table>
<thead>
<tr>
<th>Bits/Tone</th>
<th># Tones</th>
<th>Interleaver Size</th>
<th>RL + 2.5 dB</th>
<th>RL + 5 dB</th>
<th>RL + 7.5 dB</th>
<th>RL + 10 dB</th>
<th>RL + 12.5 dB</th>
<th>RL + 15 dB</th>
<th>RL + 17.5 dB</th>
<th>RL + 20 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>5,200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>10,400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,600</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td><em>15,600</em></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>58</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td><em>31,200</em></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>175</td>
<td>313</td>
<td>482</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4,800</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>11</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>10</td>
<td>24</td>
<td>115</td>
</tr>
</tbody>
</table>

* It is possible to correct all the errors with an additional 5 ms of latency.
**Next FEC in ADSL modems**

Turbo Codes (19)

- Complexity of the receiver and transmitter per tone for a log-MAP decoder

<table>
<thead>
<tr>
<th>Bit/ Tone</th>
<th># Tones</th>
<th>Interleaver Size</th>
<th>Estimates</th>
<th>Multiples</th>
<th>Add/sub</th>
<th>RAM</th>
<th>Lookups</th>
<th># Compa.</th>
<th>Precision Fixed-Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>5,200</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>374,400</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>57,600</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>28,800</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>10,400</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>748,800</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,600</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>115,200</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>6</td>
<td>6</td>
<td>7208</td>
<td>57,600</td>
<td>1536</td>
<td>2500</td>
<td>16 bits</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>15,600</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>1,123,200</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>172,800</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,200</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>86,400</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>31,200</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>2,246,400</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4,800</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>345,600</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>14</td>
<td>14</td>
<td>21624</td>
<td>172,800</td>
<td>4608</td>
<td>7500</td>
<td>16 bits</td>
</tr>
</tbody>
</table>
Next FEC in ADSL modems

Turbo Codes (20)

- Complexity of the receiver and transmitter per tone for a MAX-log-MAP decoder

<table>
<thead>
<tr>
<th>Bit/Tone</th>
<th># Tones</th>
<th>Interleaver Size</th>
<th>Estimates</th>
<th>Multiplies</th>
<th>Add/sub</th>
<th>RAM</th>
<th>Lookups</th>
<th># Compa.</th>
<th>Precision Fixed-Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>5,200</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>374,400</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>57,600</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>28,800</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>10,400</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>748,800</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,600</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>115,200</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>6</td>
<td>6</td>
<td>2088</td>
<td>57,600</td>
<td>0</td>
<td>1604</td>
<td>16 bits</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>15,600</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>1,123,200</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>172,800</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,200</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>86,400</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>31,200</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>2,246,400</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4,800</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>345,600</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,400</td>
<td>14</td>
<td>14</td>
<td>6264</td>
<td>172,800</td>
<td>0</td>
<td>4812</td>
<td>16 bits</td>
</tr>
</tbody>
</table>
**Next FEC in ADSL modems**

**Summary**

- The Turbo Code proposed is very robust against impulse noise, with up to 17.5 dB of margin for the case a spectral efficiency of 4 bits per tone and 10 dB for the case of a spectral efficiency of 12 bits per tone.
- The net coding gain is around 7.5 dB (3 dB more than with TCM), this allows, approximately, one more bit per tone. With 256 tones, this means 1.2 Mbps greater data rate than with TCM.
- Square constellations allow the use of very efficient blind equalization techniques, effectively maintaining I and Q independent.
- Gray mapping with the information bits are more protected than parity bits. Specially effective for impulse noise, because the parity does not provide information if an impulse noise is present.
- The Soft-Output Turbo decoder can provide information to the Reed-Solomon outer decoder to increase its performance by 2. This means that with the Soft-Output Turbo decoder, Reed-Solomon can correct up to R symbols.
- Fits the structure of the DMT symbols.