“Turbo Codes for xDSL modems”

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Current FEC in ADSL modems

Reed-Solomon (1)

- Actually G.992.1 & G.992.2 uses mandatory Reed-Solomon for FEC to avoid Impulse Noise.

- Reed-Solomon ensured Error Free operation for any impulse noise < than 2 DMT symbols latency < 10 ms and data rate < 6.4 Mbps with a redundancy around 12 %.

- R redundant check bytes c_0 , $c_1, \ldots, c_{R-2}, c_{R-1}$ are be appended to K message bytes $m_0, m_1, \ldots, m_{K-2}, m_{K-1}$ to form a Reed-Solomon codeword of size $N = K + R$ bytes. The check bytes are computed from the message byte using the equation:

$$C(\mathrm{D}) = M(\mathrm{D})\,\mathrm{D}^{\mathrm{R}} \text{ modulo } G(\mathrm{D})$$

$$M(\mathrm{D}) = m_0\,\mathrm{D}^{\mathrm{K}-1} + m_1\,\mathrm{D}^{\mathrm{K}-2} + \ldots + m_{\mathrm{K}-2}\,\mathrm{D} + m_{\mathrm{K}-1}$$

$$C(\mathrm{D}) = c_0\,\mathrm{D}^{\mathrm{R}-1} + c_1\,\mathrm{D}^{\mathrm{R}-2} + \ldots + c_{\mathrm{R}-2}\,\mathrm{D} + c_{\mathrm{R}-1}$$

$$G(D) = \prod_{i=0}^{R-1}\left(D + \alpha^{\mathrm{i}}\right)$$

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Current FEC in ADSL modems

Reed-Solomon (2)

Table 7-7/G.992.1 – Minimum FEC coding capabilities for ATU-C

Parameter	Fast buffer	Interleaved buffer
Parity bytes per R-S codeword	R_F = 0, 2, 4, 6, 8, 10, 12, 14, 16 (Note 1)	R_I = 0, 2, 4, 6, 8, 10, 12, 14, 16 (Notes 1 and 2)
DMT symbols per R-S codeword	S = 1	S = 1, 2, 4, 8, 16
Interleave depth	Not applicable	D = 1, 2, 4, 8, 16, 32, 64
NOTE 1 – R_F can be > 0 only if K_F > 0, and R_I can be > 0 only if K_I > 0. NOTE 2 – R_I shall be an integer multiple of S.		

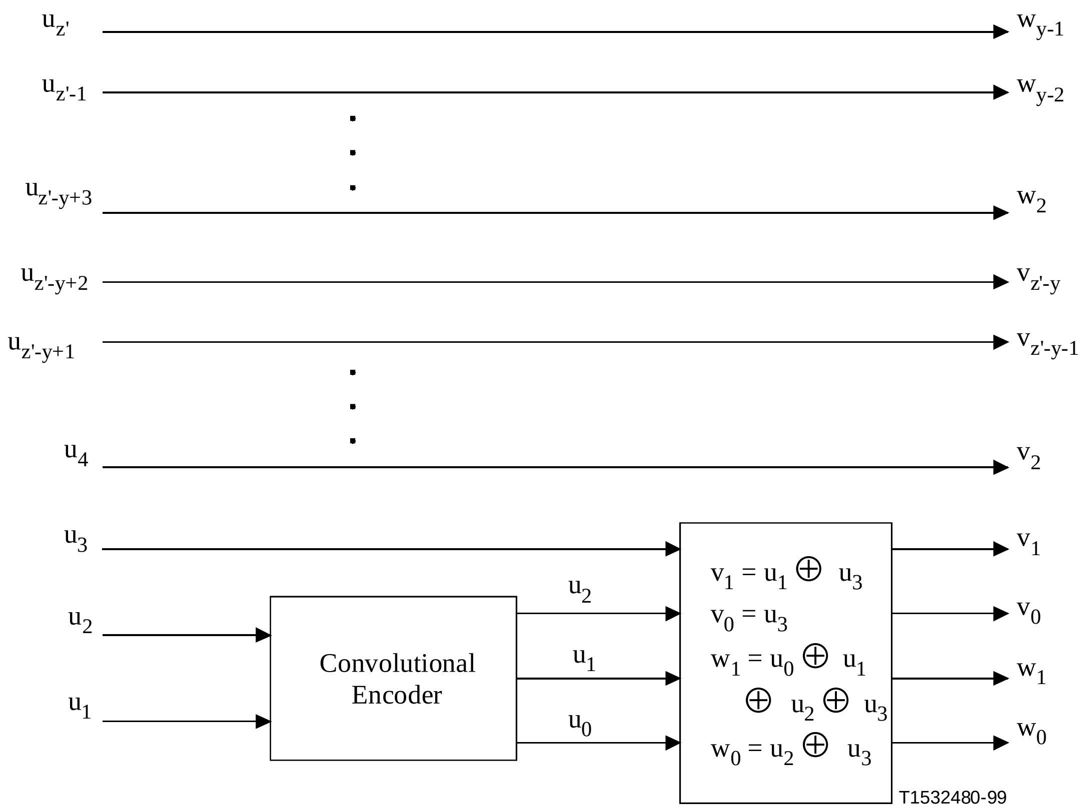
Table 8-3/G.992.1 – Minimum FEC coding capabilities for ATU-R

Parameter	Fast buffer	Interleaved buffer
Parity bytes per R-S codeword	R_F = 0, 2, 4, 6, 8, 10, 12, 14, 16 (Note 1)	R_I = 0, 2, 4, 6, 8, 10, 12, 14, 16 (Notes 1 and 2)
DMT symbols per R-S codeword	S = 1	S = 1, 2, 4, 8, 16
Interleave depth	Not applicable	D = 1, 2, 4, 8
NOTE 1 – R_F can be > 0 only if K_F > 0 and R_I can be > 0 only if K_I > 0. NOTE 2 – R_I shall be an integer multiple of S.		

Current FEC in ADSL modems

Trellis Code Modulation

- Optional in G.992.1 as inner encoder.



Next FEC in ADSL modems

- It has been agreed that the committee “shall develop a proposal for improved coding gain”.
- It is agreed that proposals for coding techniques shall present:
  - Net coding Gain and Latency for 10^{-3}, 10^{-7} and 10^{-9}.
  - Results with and without R-S as outer encoder.
  - Results for spectral efficiencies of 4 bit/s/Hz and 12 bit/s/Hz.
  - Complexity (also relative to G.992.1).
  - Impulse and AWGN noise results and error statistics.
  - Impulse noise defined as 2 DMT symbols with 5, 10 and 15 dB more noise than the Reference Level (RL).
  - Reference level is level for 10^{-7} uncoded.
  - Code rate and BER.
  - Estimation of the expected Turbo Code error floor.

Next **FEC in ADSL modems**

Turbo Codes (1)

- Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel.
- Turbo codes were first presented in the ICC 93 by C. Berrou, A. Glavieux and P. Thitimajshima.
- Turbo codes had been proposed for ADSL modems since August 1998 by VOCAL Technologies Ltd (Q4/SG15 AB-093).
- Turbo codes use at least two Parallel Concatenated Convolutional Codes that use systematic recursive codes (SRC) and iterative decoding techniques with soft-in-soft-out (SISO) decoders.
- These codes provide from the same information bit d_i, two parity bits p_i and q_i.

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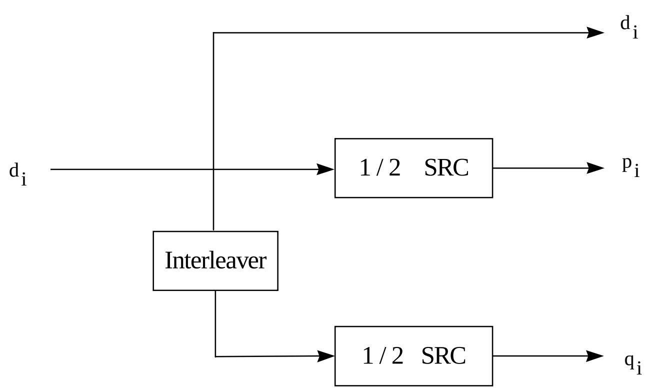
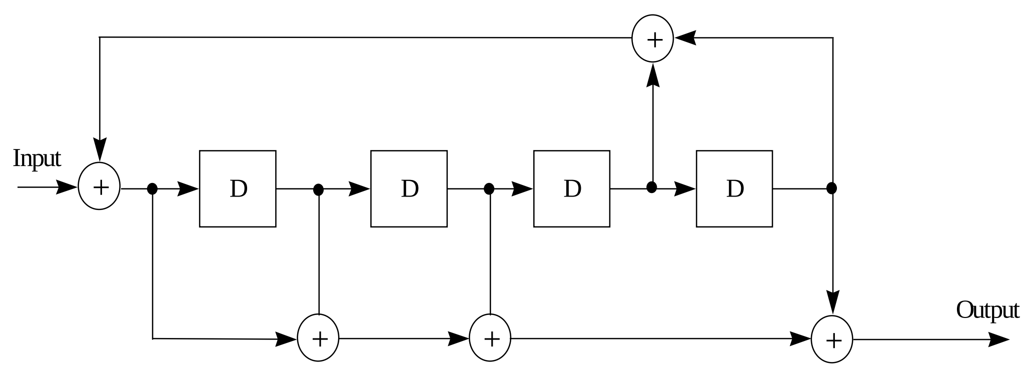
Next FEC in ADSL modems

Turbo Codes (2)

- Between the two SRC, there is an interleaver that change the order of the information bits before they enter the second SRC.
- The design of the interleaver is very important.
- It is possible to puncture (select alternately) the parity bits to reduce the redundancy with a small degradation in the performance.





Next FEC in ADSL modems

Turbo Codes (3)

- Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:

  - Inmarsat's new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
  - The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification.
  - NASA's next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
  - The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
  - The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.

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Next **FEC in ADSL modems**

Turbo Codes (4)

- *Equations:*

$$E_b / N_0[dB] = E_s / N_0\,[dB] - 10 \log_{10}(\eta)\ \ [dB]$$

$$C/N_0\ [dB\text{-}Hz] = E_b/N_0\ [dB] + 10 \log_{10}(R_b)\ \ [dB\text{-}Hz]$$

$$SNR = \frac{E\,[|a_k^2|]}{E\,[|w_k^2|]} = \frac{E\,[|a_k^2|]}{D\sigma_N^2} = \frac{E_{av}}{D\sigma_N^2}$$

$$SNR = \frac{E_s}{D\dfrac{N_0}{2}} = \frac{\eta E_b}{D\dfrac{N_0}{2}}$$

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1}$$

Next FEC in ADSL modems

Turbo Codes (5)

- Turbo code proposed by VOCAL Technologies Ltd. (BA-020 and HC-073)
- Use of square constellations that allow for use of very efficient blind equalization techniques, effectively maintaining I and Q independent.
- Gray mapping with the information bits are more protected than parity bits. This is specially effective for impulse noise, because the parity does not provide information if an impulse noise is present.
- The Soft-Output Turbo decoder can provide information to the Reed-Solomon outer decoder to increase its performance by 2. This means that with the Soft-Output turbo decoder R-S can correct up to R symbols.

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Next FEC in ADSL modems

Turbo Codes (6)

- Coding and modulation for 4 Bit/s/Hz spectral efficiency.

Puncturing and Mapping for Rate 4/6 64 QAM

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	-	q_3	-
8AM symbol (I)	(d_1, d_2, p_1)			
8AM symbol (Q)	(d_3, d_4, q_3)			
64 QAM symbol (I , Q)	$(I, Q) = (d_1, d_2, p_1, d_3, d_4, q_3)$			

Next **FEC in ADSL modems**

Turbo Codes (7)

- Coding and modulation for 4 Bit/s/Hz spectral efficiency.

$$E_{av} = (1+9+25+49)\,A^2/4 = 21\,A^2$$

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 21\,A^2\left(\frac{2 \text{ x } 2 \text{ x } E_b}{N_0}\right)^{-1} = 5.25\,A^2\left(\frac{E_b}{N_0}\right)^{-1}$$

- Bit Probabilities: The 8 AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k)$, where u_1^k is the most significant bit and u_3^k is the least significant bit . The following set can be defined.

bit-1-is-1 = $\{ A_4, A_5, A_6, A_7 \}$
bit-2-is-1 = $\{ A_0, A_1, A_6, A_7 \}$
bit-3-is-1 = $\{ A_1, A_2, A_5, A_6 \}$

$$LLR(u_n^k) = \log\left(\frac{\sum_{A_i \in \text{bit-n-is-1}} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in \text{bit-n-is-0}} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right)$$

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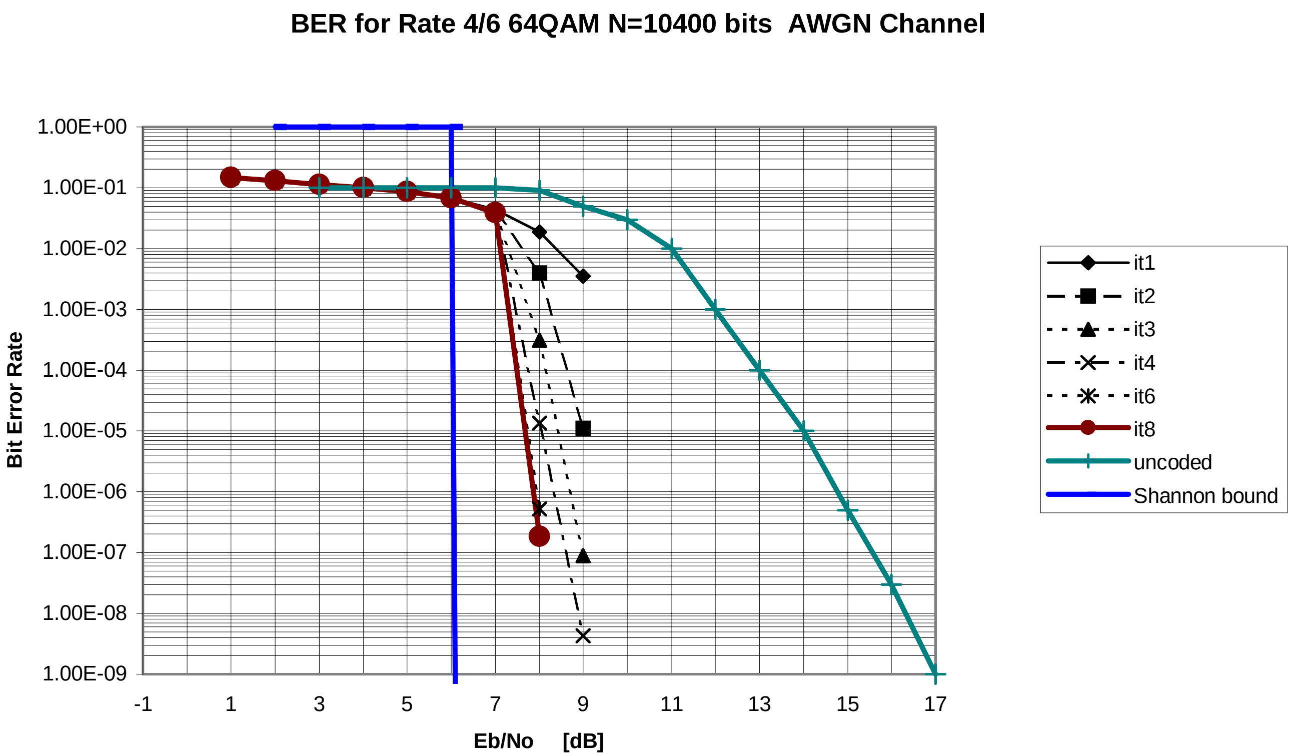
Next **FEC in ADSL modems**

Turbo Codes (8)

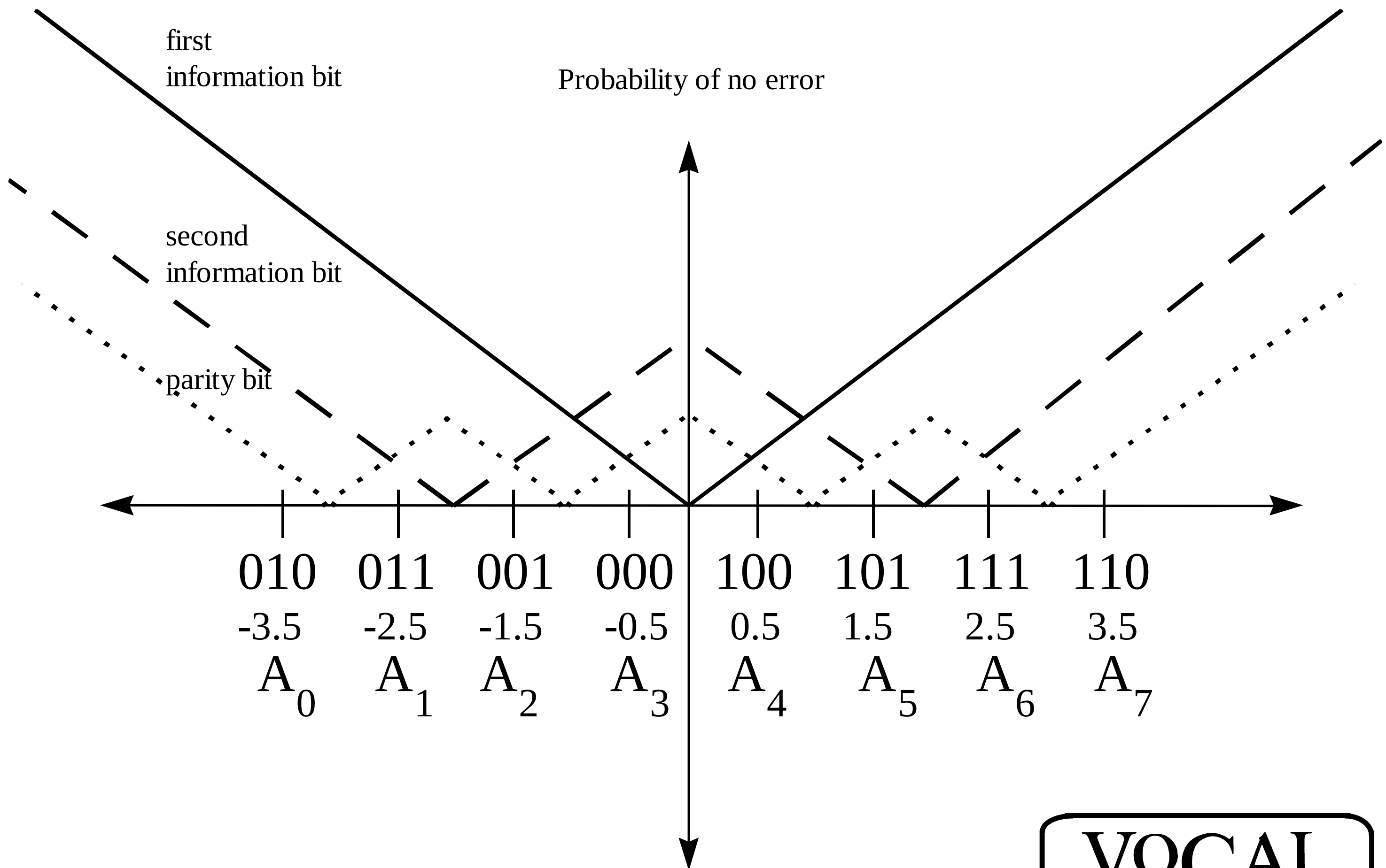
- For an interleaver size of 10,400 information bits using the interleaver defined in the 3GPP (3G TS 25.212 v3.2.0) recommendation.



***Next* FEC in ADSL modems**

Turbo Codes (9)

- Very resistance to impulse noise because of the probability of no error:



Next FEC in ADSL modems

Turbo Codes (10)

- Coding and Modulation for 12 Bit/s/Hz spectral efficiency.

Puncturing and Mapping for Rate 4/6 64 QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	D_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}
parity bit (p)	p_1	-	-	-	-	-	-	-	-	-	-	-
parity bit (q)	-	-	-	-	-	-	q_7	-	-	-	-	-
128AM symbol (I)	$(d_1, d_2, d_3, d_4 d_5, d_6, p_1)$											
128AM symbol (Q)	$(d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, q_7)$											
16384 QAM symbol (I , Q)	$(d_1, d_2, d_3, d_4 d_5, d_6, p_1, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, q_7)$											

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Next FEC in ADSL modems

Turbo Codes (11)

- Coding and modulation for 12 Bit/s/Hz spectral efficiency.

$$E_{av} = 5{,}461\, A^2$$

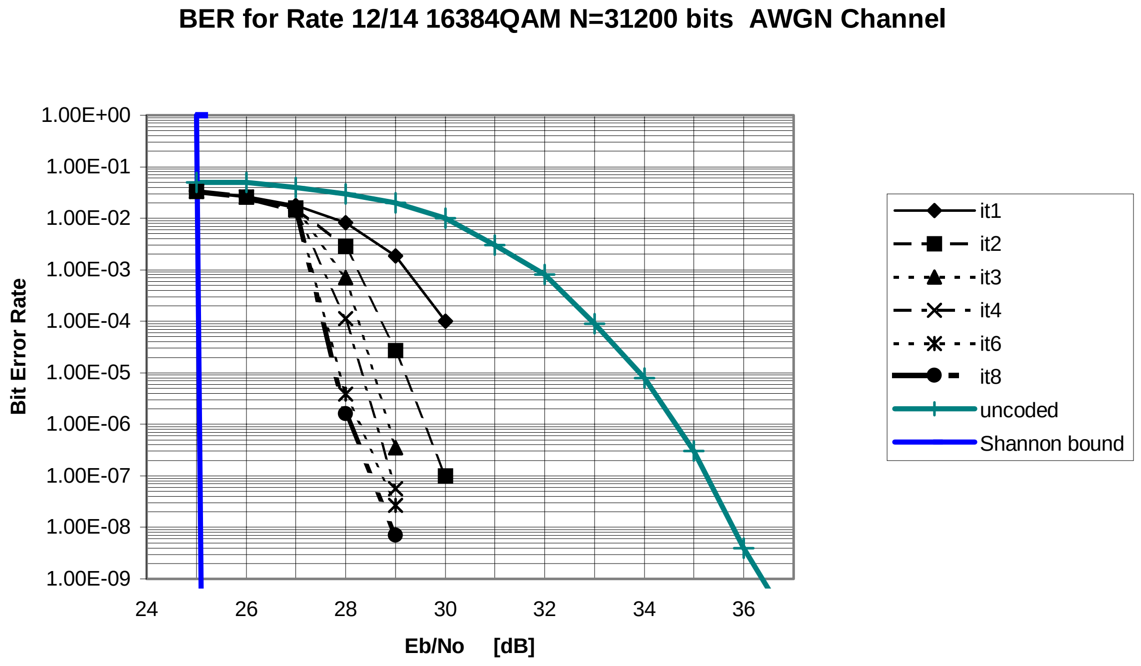
$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 5{,}461 A^2\left(\frac{2 \times 6 \times E_b}{N_0}\right)^{-1} = 455.08\, A^2\left(\frac{E_b}{N_0}\right)^{-1}$$

- Bit Probabilities: The 128 AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k, u_9^k, u_{10}^k, u_{11}^k, u_{12}^k, u_{13}^k, u_{14}^k)$ where u_1^k is the most significant bit and u_{14}^k is the least significant bit .

***Next* FEC in ADSL modems**

Turbo Codes (12)

- For an interleaver size of 31,200 information bits using the interleaver defined in the 3GPP (3G TS 25.212 v3.2.0) recommendation.



***Next* FEC in ADSL modems**

Turbo Codes (13)

- Simulation Results: Net Coding Gain without Reed-Solomon

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	4.60	7.42	7.94
		800	2	1.5	3.70	4.92	4.84
		400	1	0.7	3.30	3.62	3.84
	200	10,400	13	10.0	4.60	7.52	8.14
		1,600	2	1.5	4.10	6.42	6.64
		800	1	0.7	3.70	4.92	4.84
12	100	15,600	13	10.0	4.10	5.91	6.03
		2,400	2	1.5	3.60	5.51	5.63
		1,200	1	0.7	3.00	3.91	4.03
	200	31,200	13	10.0	4.10	6.81	7.53
		4,800	2	1.5	3.60	5.91	6.43
		2,400	1	0.7	3.60	5.51	5.63

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Next FEC in ADSL modems

Turbo Codes (14)

- Errors due to impulse noise without Reed-Solomon: The impulse noise is defined as 2 consecutive DMT symbols with an increase AWGN respect to the reference noise level of a carrier-to-noise ratio of the uncoded system.

Bits/ Tone	# Tones	Interleaver Size	RL + 2.5 dB	RL + 5 dB	RL + 7.5 dB	RL + 10 dB	RL + 12.5 dB	RL + 15 dB	RL + 17.5 dB	RL + 20 dB
4	100	5,200	0	0	0	0	0	0	0	4
		800	0	0	39	65	104	140	188	243
		400	0	0	10	50	89	127	161	214
	200	10,400	0	0	0	0	0	0	0	7
		1,600	0	0	0	127	189	267	363	448
		800	0	0	40	116	187	252	346	440
12	100	15,600	0	0	0	0	10	58	130	207
		2,400	0	0	40	78	121	171	216	295
		1,200	0	0	43	98	129	188	255	329
	200	31,200	0	0	0	0	90	175	313	482
		4,800	0	0	75	177	254	341	462	608
		2,400	0	0	80	166	244	345	457	598

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Next FEC in ADSL modems

Turbo Codes (15)

- It is interesting that for the large turbo decoders the impulse errors still tends to stay within the 2 DMT symbols. This implies a moderately large turbo coder of 5 ms follow by a convolutional interleaver/Reed-Solomon of 10 ms should create both robust performance and good impulse resistance.

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***Next* FEC in ADSL modems**

Turbo Codes (16)

- Simulation Results: Coding Gain with Reed-Solomon

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	5.00	8.62	9.64
		800	2	1.5	3.50	7.12	8.44
		400	1	0.7	3.50	6.42	7.44
	200	10,400	13	10.0	5.30	8.82	9.84
		1,600	2	1.5	4.60	7.72	8.74
		800	1	0.7	3.50	7.12	8.44
12	100	15,600	13	10.0	4.40	7.71	8.53
		2,400	2	1.5	4.60	7.41	8.33
		1,200	1	0.7	4.10	6.81	7.63
	200	31,200	13	10.0	4.40	7.71	8.53
		4,800	2	1.5	4.40	7.21	8.13
		2,400	1	0.7	4.60	7.41	8.33

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Next **FEC in ADSL modems**

Turbo Codes (17)

- Simulation Results: NET Coding Gain with Reed-Solomon

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	3.42	7.04	8.06
		800	2	1.5	1.78	5.40	6.72
		400	1	0.7	1.94	4.86	5.88
	200	10,400	13	10.0	3.72	7.24	8.26
		1,600	2	1.5	2.88	6.00	7.02
		800	1	0.7	1.94	5.56	6.88
12	100	15,600	13	10.0	0.20	3.51	4.33
		2,400	2	1.5	0.02	2.83	3.75
		1,200	1	0.7	-0.24	2.47	3.29
	200	31,200	13	10.0	1.06	4.37	5.19
		4,800	2	1.5	0.76	3.57	4.49
		2,400	1	0.7	0.26	3.07	3.99

Next FEC in ADSL modems

Turbo Codes (18)

- Errors due to Impulse noise with Reed-Solomon.

Bits/ Tone	# Tones	Interleaver Size	RL + 2.5 dB	RL + 5 dB	RL + 7.5 dB	RL + 10 dB	RL + 12.5 dB	RL + 15 dB	RL + 17.5 dB	RL + 20 dB
4	100	5,200	0	0	0	0	0	0	0	0
		800	0	0	0	0	0	0	0	0
		400	0	0	0	0	0	0	0	0
	200	10,400	0	0	0	0	0	0	0	0
		1,600	0	0	0	0	0	0	0	0
		800	0	0	0	0	0	0	0	0
12	100	*15,600	0	0	0	0	10	58	130	207
		2,400	0	0	0	0	0	0	0	0
		1,200	0	0	0	0	0	0	0	0
	200	*31, 200	0	0	0	0	90	175	313	482
		4,800	0	0	0	0	0	10	11	65
		2,400	0	0	0	0	9	10	24	115

* It is possible to correct all the errors with an additional 5 ms of latency.

Next FEC in ADSL modems

Turbo Codes (19)

- Complexity of the receiver and transmitter per tone for a log-MAP decoder

Bit/ Tone	# Tones	Interleaver Size	Estimates	Multiplies	Add/sub	RAM	Lookups	# Compa.	Precision Fixed-Point
4	100	5,200	6	6	7208	374,400	1536	2500	16 bits
		800	6	6	7208	57,600	1536	2500	16 bits
		400	6	6	7208	28,800	1536	2500	16 bits
	200	10,400	6	6	7208	748,800	1536	2500	16 bits
		1,600	6	6	7208	115,200	1536	2500	16 bits
		800	6	6	7208	57,600	1536	2500	16 bits
12	100	15,600	14	14	21624	1,123,200	4608	7500	16 bits
		2,400	14	14	21624	172,800	4608	7500	16 bits
		1,200	14	14	21624	86,400	4608	7500	16 bits
	200	31,200	14	14	21624	2,246,400	4608	7500	16 bits
		4,800	14	14	21624	345,600	4608	7500	16 bits
		2,400	14	14	21624	172,800	4608	7500	16 bits

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Next **FEC in ADSL modems**

Turbo Codes (20)

- Complexity of the receiver and transmitter per tone for a MAX-log-MAP decoder

Bit/ Tone	# Tones	Interleaver Size	Estimates	Multiplies	Add/sub	RAM	Lookups	# Compa.	Precision Fixed-Point
4	100	5,200	6	6	2088	374,400	0	1604	16 bits
		800	6	6	2088	57,600	0	1604	16 bits
		400	6	6	2088	28,800	0	1604	16 bits
	200	10,400	6	6	2088	748,800	0	1604	16 bits
		1,600	6	6	2088	115,200	0	1604	16 bits
		800	6	6	2088	57,600	0	1604	16 bits
12	100	15,600	14	14	6264	1,123,200	0	4812	16 bits
		2,400	14	14	6264	172,800	0	4812	16 bits
		1,200	14	14	6264	86,400	0	4812	16 bits
	200	31,200	14	14	6264	2,246,400	0	4812	16 bits
		4,800	14	14	6264	345,600	0	4812	16 bits
		2,400	14	14	6264	172,800	0	4812	16 bits

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Next FEC in ADSL modems

Summary

- The Turbo Code proposed is very robust again impulse noise, with up to 17.5 dB of margin for the case a spectral efficiency of 4 bits per tone and 10 dB for the case of a spectral efficiency of 12 bits per tone.
- The net coding gain is around 7.5 dB (3 dB more than with TCM), this allows, approximately, one more bit per tone. With 256 tones, this means 1.2 Mbps greater data rate than with TCM.
- Square constellations allows the use of very efficient blind equalization techniques, effectively maintaining I and Q independent.
- Gray mapping with the information bits are more protected than parity bits. Specially effective for impulse noise, because the parity does not provide information if an impulse noise is present.
- The Soft-Output Turbo decoder can provide information to the Reed-Solomon outer decoder to increase its performance by 2. This means that with the Soft-Output Turbo decoder, Reed-Solomon can correct up to R symbols.
- Fits the structure of the DMT symbols.

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