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TITLE: G.gen: New proposal of Turbo Codes for ADSL modems

ABSTRACT

This paper describes a new technique for coding and decoding signals that use turbo codes with QAM modulation for error correction

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1. Introduction:

Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Since its discovery in 1993, a lot of research has been done in the application of turbo codes in deep space communications, mobile satellite/cellular communications, microwave links, paging, in OFDM and CDMA architectures. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel. The extra coding gain offered by turbo codes can be used either to minimize bandwidth or to reduce power requirements in the link budget. Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:

- Inmarsat's new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
- The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification. The IMT-2000 represents the third generation mobile radio systems worldwide standard. The 3GPP objective is to harmonize similar standards proposals from Europe, Japan, Korea and the United States.
- NASA's next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
- The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
- The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.

In this paper we present a new perspective of the use of turbo codes for systems with different order of QAM modulations working at the same time. This perspective that avoids the most difficult part of turbo coding: the computing requirements. The way to avoid these computing requirements is to treat the QAM signal like two AM modulations (one in the I direction and one in the Q direction) and use the probabilities of the I and Q AM values as an input to the turbo code process.

Papers presented so far for ADSL modems using turbo codes give details of only 16QAM, and the computational effort is important.

Another innovation of the technique proposed here is the mapping used for the modulated signal, providing the most protected bits for the information bits and the least protected bits for the parity bits. In this paper we present the different possible puncturing for each order of a QAM modulated signal and we choose the one that provides better performance.

This is the first investigation in the performance of turbo codes for all constellation sizes, up to 1024 QAM. The recommended solutions achieve a target BER very close to the channel capacity for the respective spectral efficiency response. The turbo code schemes proposed here are more power efficient than the trellis coded modulation schemes used traditionally with V.32.bis or V.34 standards. The larger the constellation is the more improvements that this technique provides (i.e. for 16QAM the probabilities to compute in a classical turbo code are 16 points with 16 elements each point, with this technique the probabilities to compute are 8 (4 in each dimension) with 4 elements each, so if we use 10 iterations the computational savings is more than 10*(16*16/8*4) = 80 (almost two orders of magnitude); for 256QAM the computational savings is more than 10 * (256*256) / (32*16) = 1280) (over three orders of magnitude).

In the case of the G.992.1 this technique allow the system to work at the information rate bit of 400 kbps with E_b/N_0 below 2 dB (assuming 4QAM and spectral efficiency of 1bit/s/Hz 4 ksymbols/s and 100 tones).

We propose the use of square QAM constellations with different puncturing values as function of the signal-to-noise ratio. The square constellations allow for use of very efficient blind equalization techniques an effective maintaining I and Q independent. Taking into account the values of bi and gi we decide which constellation to use and with which parity.

2. Capacity Bounds

The minimum E_b/N_0 values to achieve the Shannon bound, 4QAM, 8QAM, 16QAM, 32QAM, 64QAM, 128QAM, 256QAM, 512QAM and 1024QAM bounds for spectral efficiencies from 2/3 up to 7 bits/s/Hz respectively are as in Table 1 for a BER=10^{-5}.

Table 1. Shannon and QAM bounds.

Spectral efficiency η [bit/s/Hz]	**Shannon bound [dB]**	**4 QAM bound [dB]**	**16 QAM bound [dB]**	**64 QAM bound [dB]**	**256 QAM bound [dB]**	**1024 QAM bound [dB]**
2/3	-0.5	0.3	-0.4	-0.4	-0.4	-0.4
1	0	1.0	0.1	0.1	0.1	0.1
2	1.75	∞	2.1	2.09	2.09	2.09
3	3.7	-	4.6	4.3	4.3	4.3
4	5.6	-	∞	6.6	6.6	6.6
5	7.9	-	-	9.1	9.0	9.0
6	10.3	-	-	∞	11.7	11.7
7	12.6	-	-	-	14.5	14.5

The conversion from E_s / N_0 to E_b / N_0 is performed using the following relation

$$E_b / N_0[dB] = E_s / N_0\,[dB] - 10 \log_{10}(\eta)\ \ [dB] \qquad (1)$$

where η is the number of information bits per symbol.

The required C/N_0 given a certain E_b/N_0 can be found using the following relation:

$$C/N_0\ [dB\text{-}Hz] = E_b/N_0\ [dB] + 10 \log_{10}(R_b)\ \ [dB\text{-}Hz] \qquad (2)$$

where R_b is the information bit rate.

For a D-dimension modulation the following formulae are used:

$$SNR = \frac{E[|a_k^2|]}{E[|w_k^2|]} = \frac{E[|a_k^2|]}{D\sigma_N^2} = \frac{E_{av}}{D\sigma_N^2} \qquad (3)$$

$$SNR = \frac{E_s}{D\frac{N_0}{2}} = \frac{\eta E_b}{D\frac{N_0}{2}} \qquad (4)$$

where σ^2_N is the noise variance in each of the D dimension and η is the number of information bits per symbol. From the above relations:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} \quad (5)$$

3. Coding

The proposed coding scheme is shown in Figure 1.The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

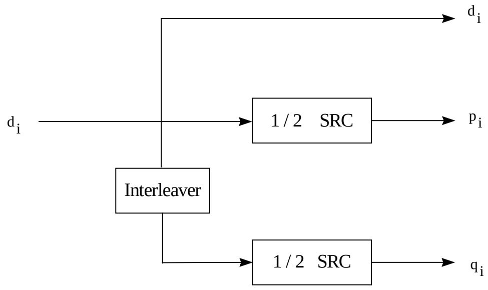

Figure 1

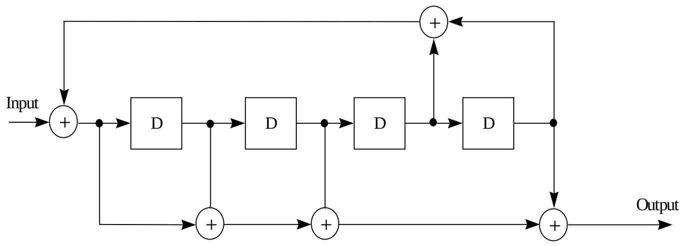

Figure 2

4. Interleaver design.

The interleaver proposed in this paper is the one defined in NT-122. It is an even-odd smile interleaver, where the number of rows must be the number of columns plus one. For example, an interleaver of 2100 information bits long, the input bit can be arranged in a 46x47 matrix. The interleaver output is in a diagonal from left to right, down to up.

```
#define MAX_CINDEX 46               /* Number of columns in the array */
#define MAX_RINDEX 47               /* Number of rows in the array */
#define MAX_ELEMENT 2100            /* Matches block length (a multiple of DMT symbols) */

void main (void)
{
      int ra, ca;                   /* Ia sequence row and column indices */
      int count;                    /* Counter for each bit in DMT frame */
      int element;                  /* Element number used for finding if element within array */

      /* Initial sequence indices */
      ra = MAX_RINDEX - 1;
      ca = 0;
      /* Adjust the initial indices for Ia if beyond ending element */
      element = ra * MAX_CINDEX + ca;
      while (element >= MAX_ELEMENT) {
            ra--;
            ca++;
            if (ra < 0) {
                  ra = MAX_RINDEX - 1;
                  ca = ca + (MAX_RINDEX - 1);
            }
            ca = ca % MAX_CINDEX;
            element = ra * MAX_CINDEX + ca;
      }
      /* Fetch all elements in sequence Ia */
      for (count = 0; count < MAX_ELEMENT; count++) {
            /* Fetch array[ra][ca] */
            /* Update indices for next access */
            do {
                  ra--;
                  ca++;
                  if (ra < 0) {
                        ra = MAX_RINDEX - 1;
                        ca = ca + (MAX_RINDEX - 1);
                  }
                  ca = ca % MAX_CINDEX;
                  element = ra * MAX_CINDEX + ca;
            } while (element >= MAX_ELEMENT);
      }
}
```

5. Modulation For 1 Bit/s/Hz Spectral Efficiency.

The scheme proposed in this case combines a rate 1/2 coding scheme with 4QAM.

5.1 Puncturing

In order to obtain a code rate of 1/2, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 2.

Table 2. Puncturing and Mapping for Rate 1/2 4QAM.

Information bit (d)	d_1	d_2
Parity bit (p)	p_1	-
Parity bit (q)	-	q_2
2AM symbol (I)	$(u_1) = (d_1)$	$(u_1) = (d_2)$
2AM symbol (Q)	$(u_2) = (p_1)$	$(u_2) = (q_2)$
4QAM symbol (I , Q)	$(I,Q) = (u_1, u_2) = (d_1 , p_1)$	$(I,Q) = (u_1, u_2) = (d_2 , q_2)$

5.2.Modulation

A 4QAM scheme is shown in Figure 3. An equivalent 2AM modulation is shown in Figure 4.

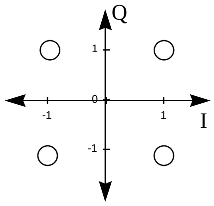

Figure 3

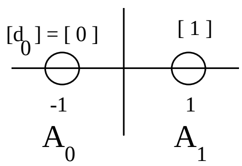

Figure 4

At time k, the symbol $u^k = (u_1^k, u_2^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 4QAM constellation with points at -A and A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2)}{4} = 2A^2 \qquad (6)$$

For a rate 1/24 code and 4QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right) = 2A^2\left(\frac{2 \text{ x } 1 \text{ x } E_b}{N_0}\right)^{-1} = A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (7)$$

5.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{1}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1i}^k)^2]}{\sum_{i=1}^{1}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2]}\right) \qquad (8)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{1}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{1i}^k)^2]}{\sum_{i=1}^{1}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{0i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_1)^2]}{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_0)^2]}\right) \qquad (9)$$

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 4 LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 2 bit-LLR values are reduced to one term each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non-punctured scheme.

5.4 Simulations Results.

Figure 5 shows the performance of a turbo code using a 1024 bit S-type interleaver. The target BER of 10^{-7} for a 1,024 information bit interleaver can be achieve at $E_b/N_0 = 2.1$ dB

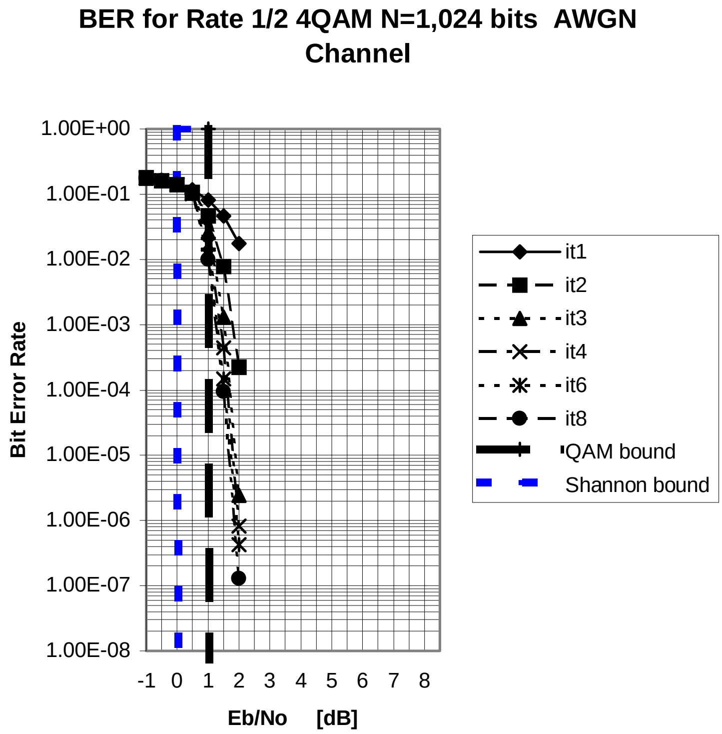

Figure 5

6. Coding And Modulation For 2 Bit/S/Hz Spectral Efficiency.

The investigated combines 2/4 coding scheme with 16QAM.

6.1 Puncturing.

In order to obtain a rate 2/4 code, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 3.

Table 3. Puncturing and Mapping for Rate 2/4 16QAM.

Information bit (d)	d_1	d_2
parity bit (p)	p_1	-
parity bit (q)	-	q_2
4AM symbol (I)	$(u_1, u_2) = (d_1, p_1)$	
4AM symbol (Q)	$(u_3, u_4) = (d_2, p_2)$	
16 QAM symbol (I , Q)	$(I, Q) = (u_1, u_2, u_3, u_4) = (d_1, p_1, d_2, p_2)$	

6.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 16QAM constellation with points at -3A, -A,A and 3A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2) + 8(A^2 + 9A^2) + 4(9A^2 + 9A^2)}{16} = 10A^2 \qquad (10)$$

For a rate 1/2 code and 16QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right) = 10A^2\left(\frac{2 \times 2 \times E_b}{N_0}\right)^{-1} = 2.5A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (11)$$

It is assumed that the time k, u_1^k and u_2^k modulate the I component and u_3^k and u_4^k modulate the Q component for a 16QAM scheme. The symbol u^k symbol has the following mapping: $u^k = (u_1^k, u_2^k, u_3^k, u_4^k) = (d^i, p^i, d^{i+1}, q^{i+1})$. The parity bits are mapped to the least protected bits of the QAM symbol. Note that k denotes the symbol time index and i the information bit time index. This means a puncturing of one in two parity bits. Considering two independent Gaussian noise sources with identical variance σ^2_N, the LLR can be determined independently for each I and Q.

At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the 4 bit-LLR values. The mapping of the information bit is made to the most protected bit in each dimension (u_1^k for the I signal and u_3^k for the Q signal). In order to estimate the performance of this scheme, rate 1/2 turbo code and 4AM modulation is used, instead of 16QAM modulation.

6.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2]}\right) \qquad (12)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2]}\right) \qquad (13)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_2)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_1)^2]}\right) \qquad (14)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_2)^2]}\right) \qquad (15)$$

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 16LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 4 bit-LLR values are reduced to 2 terms each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non punctured scheme.

6.4 Simulations Results.

The rate 2/4 16QAM scheme described in this chapter achieves the target BER at less than 1.5 dB from capacity. The implementation in hardware is feasible and it can be used at very high data rates. The target BER of 10^{-7} can be achieved at E_b/N_0 = 4.5 dB for N=1024 information bits, as shown in Figure 6.

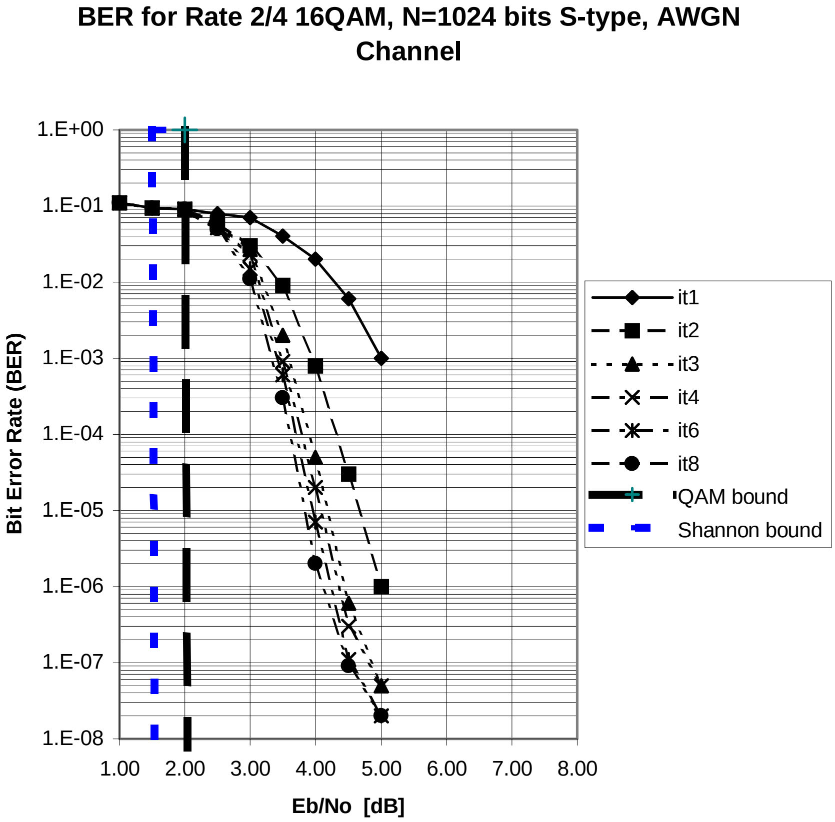

Figure 6

7. Coding And Modulation For 3 Bit/S/Hz Spectral Efficiency.

Two options are investigated in this section. The first scheme combines a rate 3/4 coding scheme with 16QAM. The second scheme combines a rate 3/4 coding scheme with 64QAM.

7.1 Option 1 Rate 3/4 Turbo code and 16QAM

7.1.1 Puncturing.

In order to obtain a rate 3/4 code, the puncturing pattern given in Table 4 is used.

Table 4. Puncturing and Mapping for Rate 3/4 16QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	-	p_2	-	-	-	-
parity bit (q)	-	-	-	-	q_5	-
4AM symbol (I)	(d_1, d_2)			(d_4, d_5)		
4AM symbol (Q)	(d_3, p_2)			(d_6, p_5)		
16 QAM symbol (I ,Q)	$(I, Q) = (d_1, d_2, d_3, p_2)$			$(I, Q) = (d_4, d_5, d_6, q_5)$		

7.1.2 Modulation

It is assumed that at time k, u_1^k and u_2^k modulate the I component and u_3^k and u_4^k modulate the Q component of a 16QAM scheme. In order to estimate the performance of this scheme, rate 3/4 turbo codes and 4 AM modulation are used. For a rate 3/4 code and 16QAM, noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 10\, A^2\left(\frac{2x3x\, E_b}{N_0}\right)^{-1} = \frac{10}{6}A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (16)$$

The puncturing and mapping scheme is shown in Table 4 for 6 consecutive information bits that are encoded into 8 coded bits, therefore two 16QAM symbols.

7.1.3 Bit Probabilities

For each received symbol, the bit probabilities are computed as described in equations (12)-(15).

7.1.4 Simulation Results

Figure 7 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 5.75 dB for N = 6144 information bits.

7.2 Option 2 Rate 3/6 Turbo code and 64QAM

7.2.1 Puncturing.

In order to obtain a rate 3/6 code, the puncturing pattern used is given in Table 5.

Table 5. Puncturing and Mapping for Rate 3/6 64QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	p_1	-	p_3	-	p_5	-
parity bit (q)	-	q_2	-	q_4	-	q_6
8AM symbol (I)	(d_1, d_2, p_1)			(d_4, d_5, q_4)		
8AM symbol (Q)	(d_3, p_3, q_2)			(d_6, p_5, q_6)		
64 QAM symbol (I , Q)	$(I,Q)=(d_1,d_2, p_1,d_3, p_3,q_2)$			$(I,Q)= (d_4,d_5, q_4,d_6, p_5,q_6)$		

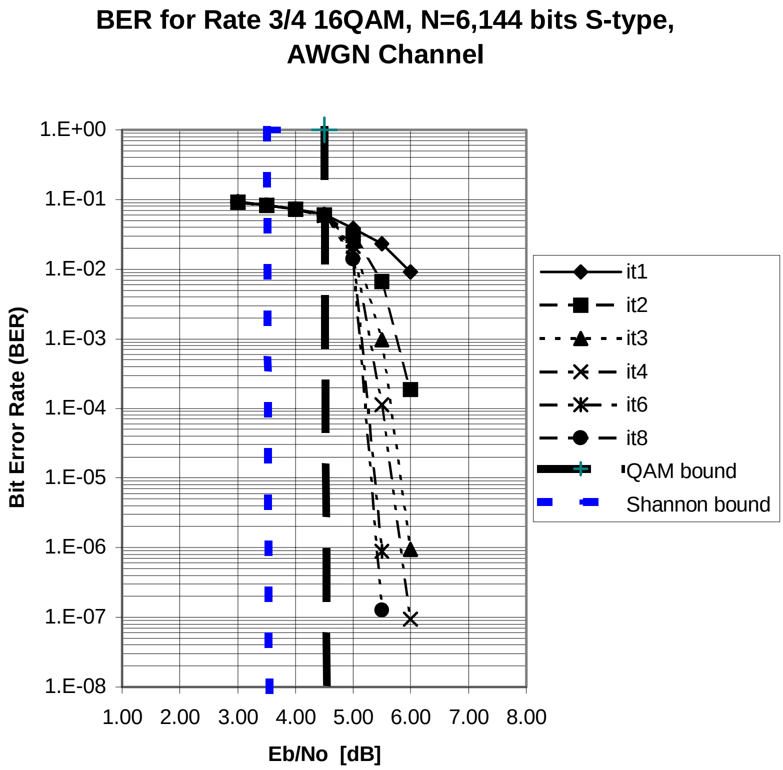

Figure 7

7.2.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k)$, is sent through the channel and the point r^k in two dimensional space is received. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of a 64QAM scheme. For 64QAM constellations with points at -7A, -5A, -3A, -A, A, 3A, 5A, 7A The E_{av} is:

$$E_{av} = (8(49+25+9+1)+8(25+49+49+9+49+1)+8(25+9+25+1)+8(9+1))\,A^2 / 64 = 42\,A^2 \qquad (17)$$

For a rate 3/6 code and 64QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 42\,A^2\left(\frac{2x3x\,E_b}{N_0}\right)^{-1} = 7\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (18)$$

In order to estimate the performance of this scheme, when rate 3/6 turbo codes and 8AM modulation is used. The puncturing and mapping scheme is shown in Table 5 for 6 consecutive information bits that are encoded into 12 coded bits, therefore two 64QAM symbols.

7.2.3 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for the I dimension:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \qquad (19)$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_4)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_5)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_6)^2] + \exp[-\frac{1}{2\sigma_n^2}(I^k - A_7)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2 + \exp[\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}\right)$$

$$LLR(u_2^k) \;=\; \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) =$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2]}\right) \tag{20}$$

$$LLR(u_3^k) \;=\; \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) =$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2]}\right) \tag{21}$$

An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

7.2.4 Simulation Results

Figure 8 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 6.1 dB for N = 6,144 information bits. This result is 0.5 dB worse than the performance of the rate 3/4 16QAM scheme.

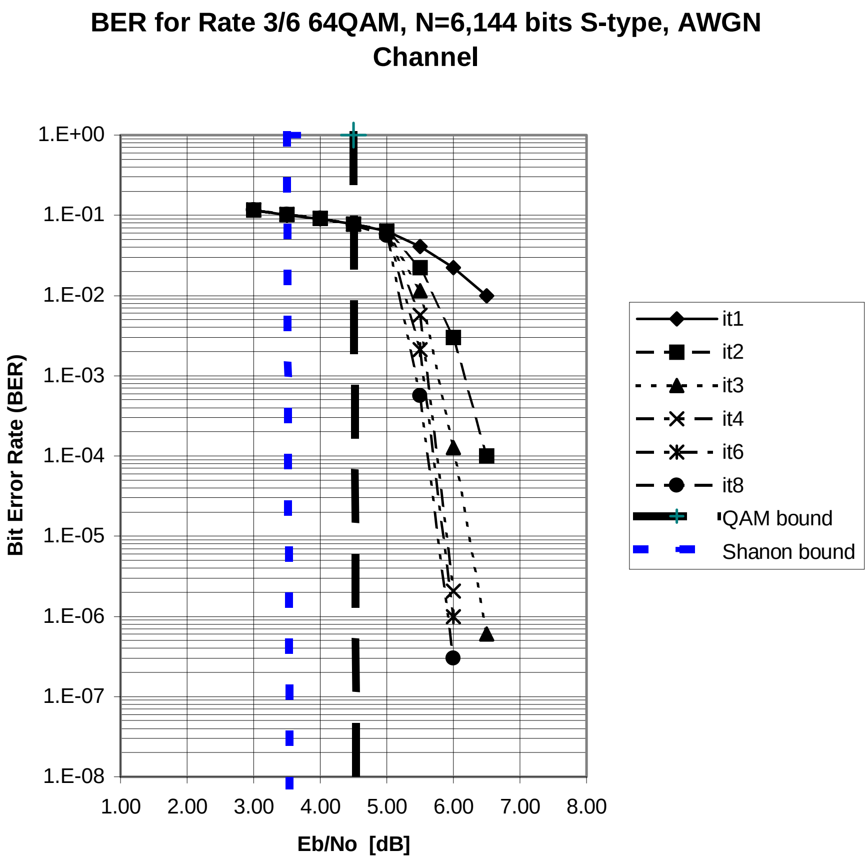

Figure 8

8. Coding And Modulation For 4 Bit/S/Hz Spectral Efficiency.

This section investigated four schemes for transmission of 4 information bits in a 64 QAM symbol. The mapping used for 64QAM constellation has a very significant impact on the performance of these schemes. The first scheme uses independent I and Q mapping and also uses Gray mapping in each dimension. The second scheme uses independent I and Q mapping, but natural mapping in each dimension. The third scheme used a conventional trellis coded modulation approach based on Ungerboeck set partitioning of the 64QAM set. This partitioning technique splits the constellation in sub-constellations with increased Euclidian distance. In these schemes all the information bits are coded. The fourth scheme used the same conventional trellis coded modulation approach based on Ungerboeck set partitioning of the 64QAM set. However, only two information bits are encoded by a rate 1/2 code. The four coded bits select a subpartition of four points. The other two information bits, which are sent uncoded, identify the transmitted point.

8.1 Option 1 – Rate 4/6 64QAM with independent I and Q and with Gray Mapping.

8.1.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing pattern used is shown in Table 6.

Table 6. Puncturing and Mapping for Rate 4/6 64QAM Option 1

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	-	q_3	-
8AM symbol (I)	(d_1, d_2, p_1)			
8AM symbol (Q)	(d_3, d_4, q_3)			
64 QAM symbol (I , Q)	$(I, Q)=(d_1,d_2, p_1,d_3, d_4,q_3)$			

8.1.2 Modulation

Gray mapping was used in each dimension. Four information bits are required to be sent using a 64QAM constellation. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 42\,A^2\left(\frac{2 \text{ x } 4 \text{ x } E_b}{N_0}\right)^{-1} = 5.25\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (22)$$

The puncturing and mapping scheme is shown in Table 6 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 6 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ^2_N, the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

8.1.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) = \tag{23}$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2]}\right)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) = \tag{24}$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2]}\right)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) = \tag{25}$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2]}\right)$$

For I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

8.1.4 Simulation Results

Figure 9 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 8.3 dB.

8.2 Option 2 – Rate 4/6 64QAM with independent I and Q and Natural Mapping.

8.2.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing pattern used is shown in Table 7.

Table 7. Puncturing and Mapping for Rate 4/6 64QAM Option 2

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	-	q_3	-
8AM symbol (I)	(d_1, d_2, p_1)			
8AM symbol (Q)	(d_3, d_4, q_3)			
64 QAM symbol (I , Q)	$(I, Q)=(d_1, d_2, p_1, d_3, d_4, q_3)$			

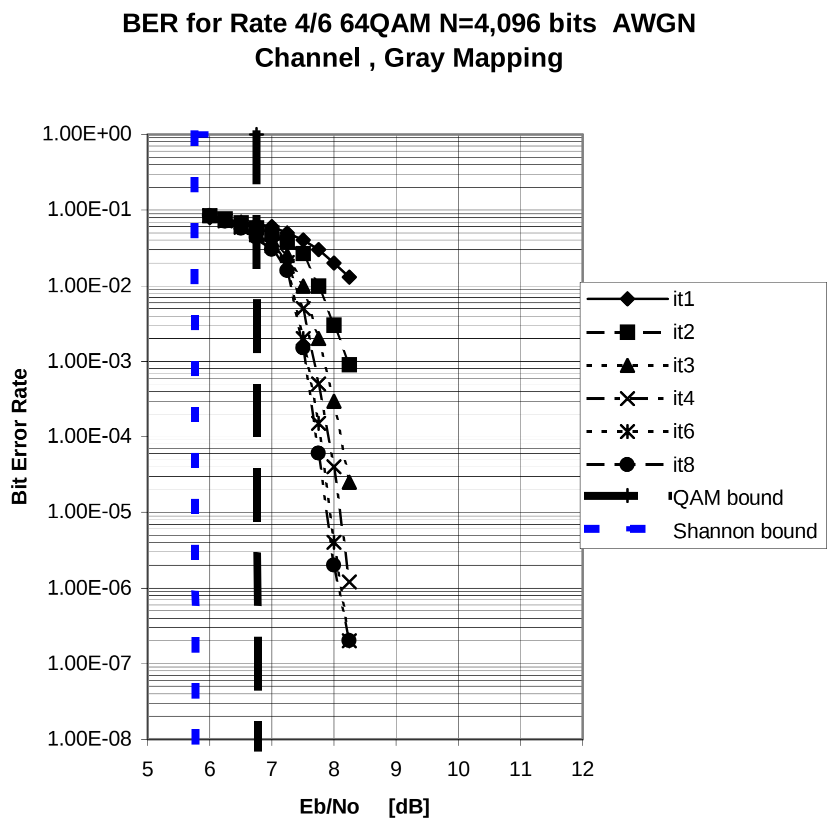

Figure 9.

8.2.2 Modulation

Natural mapping is used in each dimension. Four information bits are required to be sent using a 64QAM constellation. This is equivalent to a rate 2/3 coding scheme. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 42\, A^2\left(\frac{2x4x\, E_b}{N_0}\right)^{-1} = 5.25\, A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (26)$$

The puncturing and mapping scheme is shown in Table 7 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 7 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ^2_N, the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

8.2.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as described in equations (23) (24) and (25) for I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate LLR(u_4^k), LLR(u_5^k) and LLR(u_6^k).

8.2.4 Simulation Results

Figure 10 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 10.5 dB.

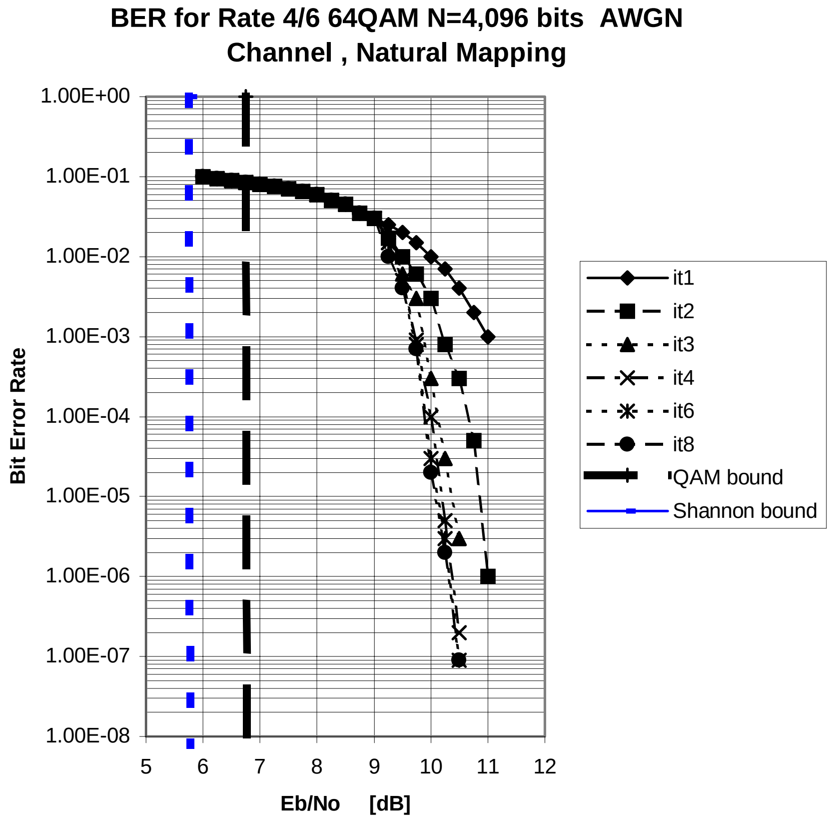

Figure 10

8.3 Option 3 – Trellis Coded Modulation with 4 bits coded.

8.3.1 Coding

In this scheme, all four information bits are coded by a rate 4/6 code. Only two parity bits are transmitted. The six bits produced select a point in the 64 QAM constellation. The proposed coding scheme is shown in Figure 1. The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

8.3.2 Puncturing

In order to obtain a rate 4/6 code, the puncturing patter used is shown in Table 8.

Table 8. Puncturing and Mapping for Rate 4/6 64QAM Option 3

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	-	q_3	-
64 QAM symbol (I , Q)	(I ,Q)=(d_1,d_2, d_3, d_4, p_1 , q_3)			

8.3.3 Modulation

A trellis coded modulation scheme is employed. The six bits divide the 64 QAM constellation based on increased Euclidean distance. The puncturing and mapping scheme is shown in Table 8 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol.

8.3.4 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for each received symbol before the turbo decoding process can start.

$$LLR(u_1^k) = \log\left(\frac{\sum_{u_1^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_1^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (27)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{u_2^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_2^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (28)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{u_3^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_3^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (29)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{u_4^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_4^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (30)$$

$$LLR(u_5^k) = \log\left(\frac{\sum_{u_5^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_5^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (31)$$

$$LLR(u_6^k) = \log\left(\frac{\sum_{u_6^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_6^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (32)$$

The $\| R^k - P_i \|$ represents the squared Euclidian distance between the received point R^k at the time k and a constellation point P_i.

8.3.5 Simulation Results

This scheme required much higher computational effort than previous options and would be difficult to implement in hardware. Figure 11 shows the simulation results for 4,096 information bits using S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 11.5 dB.

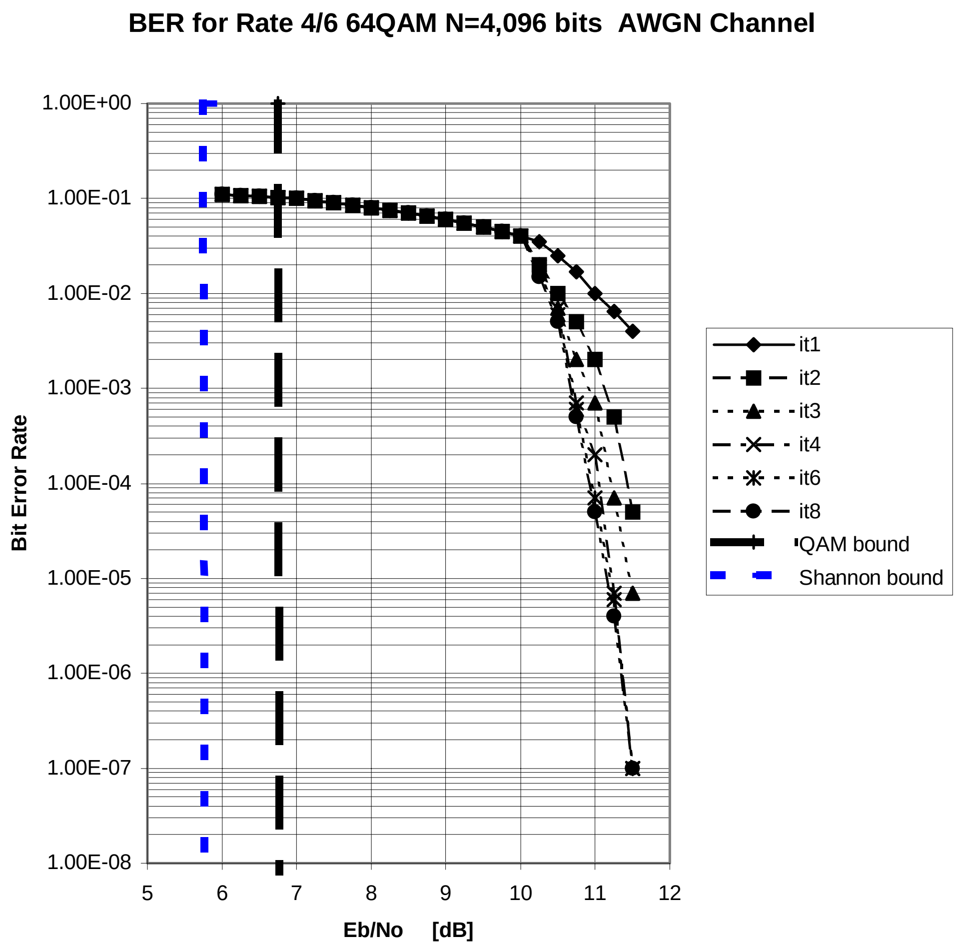

Figure 11

8.4 Option 4 – Trellis Coded Modulation with 2 bits coded.

8.4.1 Coding

In this scheme, only two information bits are coded by a rate 4/8 code. The other two information bits are sent uncoded. The four coded bits (two information bits plus two parity bits) selects a four point constellation (16 constellations in total) and the two uncoded bits select a point in the constellation (four points in each constellation). The proposed coding scheme is shown in Figure 1. The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

8.4.2 Puncturing

In order to obtain a rate 4/6 code, every other parity bit is punctured as shown in Table 9.

Table 9. Puncturing and Mapping for Rate 4/6 64QAM Option 4

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	q_2	-	-
64 QAM symbol (I , Q)	$(I ,Q) = (d_1, p_1, d_3, d_2, q_2 , d_4)$			

8.4.3 Modulation

A trellis coded modulation scheme is employed. The four coded bits divide the 64QAM constellation based on increased Euclidean distance. The 64QM constellation is partitioned by the four coded bits in 16 subsets with four point each. The two uncoded bits will select one of the four points of the subset. Each 16 points constellation subset can now be further partitioned. The puncturing and mapping scheme is shown in Table 9 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol.

8.4.4 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for each received symbol before the turbo decoding process can start.

$$LLR(u_1^k) = \log\left(\frac{\sum_{u_1^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_1^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (33)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{u_2^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_2^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (34)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{u_3^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_3^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (35)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{u_4^k=1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_i \|\right)}{\sum_{u_4^k=0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - P_j \|\right)}\right) \quad (36)$$

each summation in equations (33) (34) (35) and (36) is over 32 points

The $\| R^k - P_i \|$ represents the squared Euclidian distance between the received point R^k at the time k and a constellation point P_i.

8.4.5 Simulation Results

Figure 12 shows the simulation results for 4,096 bit S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 11.5$ dB.

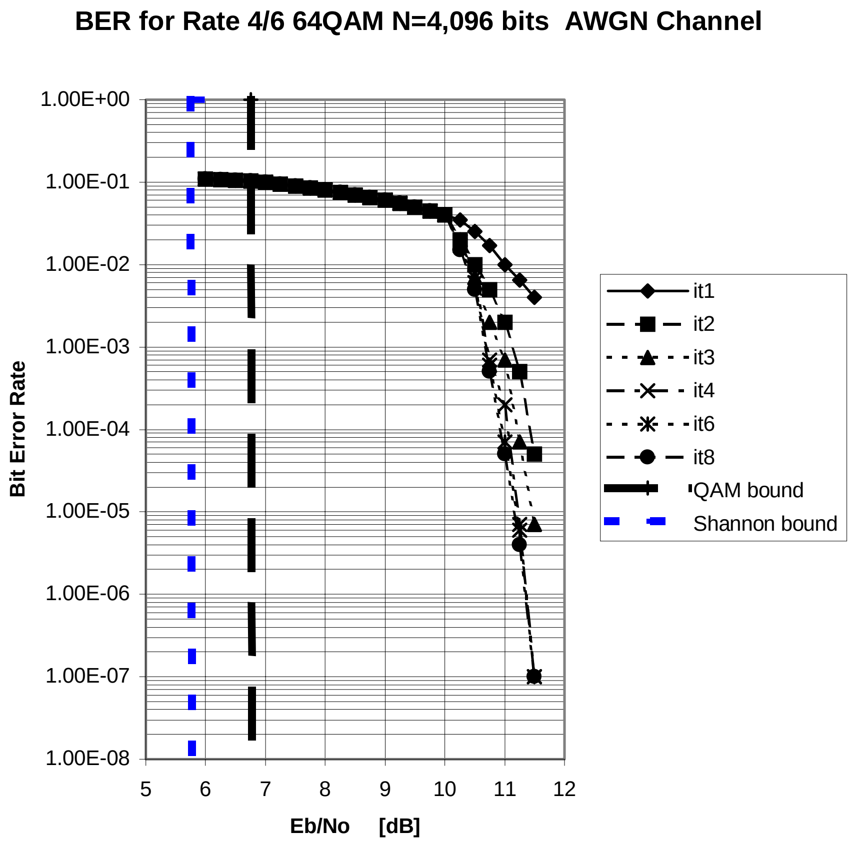

Figure 12.

9. Coding And Modulation For 5 Bit/S/Hz Spectral Efficiency.

This section investigated a rate 5/8 coding scheme with 256QAM.

9.1 Puncturing

In order to obtain a rate 5/8 code, the puncturing pattern used is shown in Table 10.

Table 10. Puncturing and Mapping for Rate 5/8 256QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
parity bit (p)	p_1	-	-	-	p_5	-	-	p_8	-	-
parity bit (q)	-	-	q_3	-		q_6	-	-	-	q_{10}
16AM symbol (I)	(d_1, d_2, d_3, p_1)					(d_6, d_7, d_8, q_6)				
16AM symbol (Q)	(d_4, d_5, q_3, p_5)					$(d_9, d_{10}, p_8, q_{10})$				
256 QAM symbol (I , Q)	$(d_1, d_2, d_3, p_1, d_4, d_5, q_3, p_5)$					$(d_6, d_7, d_8, q_6, d_9, d_{10}, p_8, q_{10})$				

9.2 Modulation

For a 256QAM constellation with points at –15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A. E_{av} is:

$$E_{av} = 170\,A^2 \qquad (37)$$

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k , u_3^k and u_4^k modulate the I component and u_5^k , u_6^k , u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 5/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 170\,A^2\left(\frac{2 \; x \; 5 \; x \, E_b}{N_0}\right)^{-1} = 17\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (38)$$

In order to study the performance of this scheme, a rate 5/8 turbo code and a 16AM is used. The 256QAM scheme will achieve a similar performance in terms of bit error rate (BER) at twice the spectral efficiency, assuming an ideal demodulator. The puncturing and mapping scheme shown in Table 10 is for 10 consecutive information bits that are coded into 16 encoded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 5/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 5 bits/s/Hz.

9.3 Bit Probabilities

The 16AM symbol is defined as $u^k= (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit . The following set can be defined.

1. bit-1-is-0 = { A_0 ,A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 }
2. bit-1-is-1 = { A_8 ,A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15} }
3. bit-2-is-0 = { A_0 ,A_1 , A_2 , A_3 , A_8 , A_9 , A_{10} , A_{11} }
4. bit-2-is-1 = { A_4 ,A_5 , A_6 , A_7 , A_{12} , A_{13} , A_{14} , A_{15} }
5. bit-3-is-0 = { A_0 ,A_1 , A_4 , A_5 , A_8 , A_9 , A_{12} , A_{13} }
6. bit-3-is-1 = { A_2 ,A_3 , A_6 , A_7 , A_{10} , A_{11} , A_{14} , A_{15} }
7. bit-4-is-0 = { A_0 ,A_2 , A_4 , A_6 , A_8 , A_{10} , A_{12} , A_{14}}
8. bit-4-is-1 = { A_1 ,A_3 , A_5 , A_7 , A_9 , A_{11} , A_{13} , A_{15} }

From each received symbol, R^k , the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}1\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}1\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \qquad (39)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}2\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}2\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \qquad (40)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}3\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}3\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \qquad (41)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}4\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}4\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \qquad (42)$$

9.4 Simulation Results

Figure 13 shows the simulation results for 5,120 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 11.8 dB.

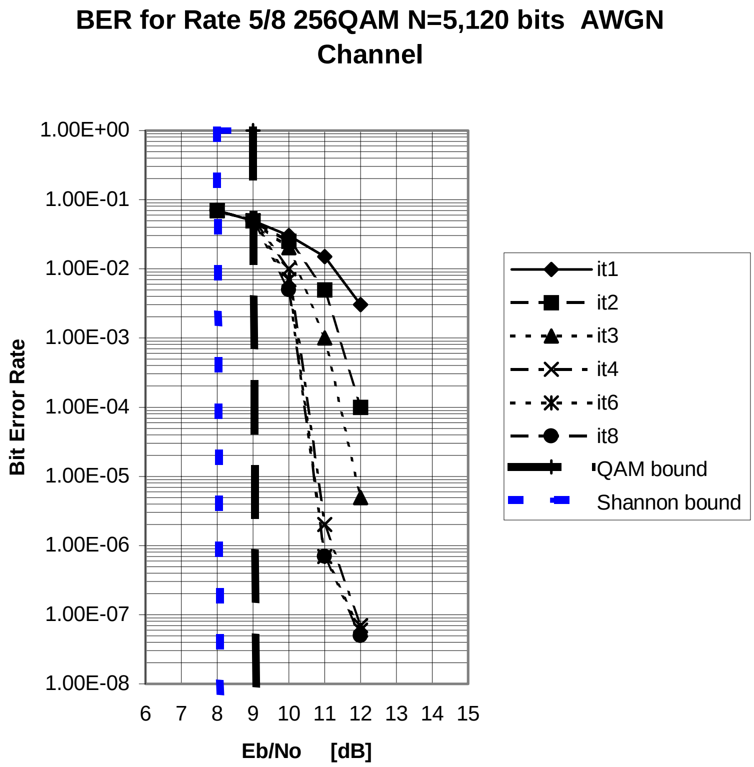

Figure 13

10. Coding And Modulation For 6 Bit/Hz Spectral Efficiency

This section investigates a rate 6/8 coding scheme with 256QAM.

10.1 Puncturing

In order to obtain a rate 6/8 code, the puncturing pattern used is shown in Table 11.

Table 11. Puncturing and Mapping for Rate 6/8 256QAM Option 1

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	p_1	-	-	-	-	-
parity bit (q)	-	-	-	q_4	-	-
16AM symbol (I)	(d_1, d_2, d_3, p_1)					
16AM symbol (Q)	(d_4, d_5, d_6, q_4)					
256QAM symbol (I, Q)	$(I, Q) = (d_1, d_2, d_3, p_1, d_4, d_5, d_6, q_4)$					

10.2 Modulation

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k and u_4^k modulate the I component and u_5^k, u_6^k, u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 6/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 170\,A^2\left(\frac{2\ x\ 6\ x\ E_b}{N_0}\right)^{-1} = 14.16\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (43)$$

The puncturing and mapping scheme shown in Table 11 is for 6 consecutive information bits that are coded into 8 coded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 6/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 6 bits/s/Hz.

10.3 Bit Probabilities

The 16AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = { A_0 ,A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 }
2. bit-1-is-1 = { A_8 ,A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15} }
3. bit-2-is-0 = { A_0 ,A_1 , A_2 , A_3 , A_8 , A_9 , A_{10} , A_{11} }
4. bit-2-is-1 = { A_4 ,A_5 , A_6 , A_7 , A_{12} , A_{13} , A_{14} , A_{15} }
5. bit-3-is-0 = { A_0 ,A_1 , A_4 , A_5 , A_8 , A_9 , A_{12} , A_{13} }
6. bit-3-is-1 = { A_2 ,A_3 , A_6 , A_7 , A_{10} , A_{11} , A_{14} , A_{15} }
7. bit-4-is-0 = { A_0 ,A_2 , A_4 , A_6 , A_8 , A_{10} , A_{12} , A_{14}}
8. bit-4-is-1 = { A_1 ,A_3 , A_5 , A_7 , A_9 , A_{11} , A_{13} , A_{15} }

From each received symbol, R^k, the bit probabilities are computed as equations (39) to (42).

10.4 Simulation Results

Figure 14 shows the simulation results for 6,144 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 14.2 dB.

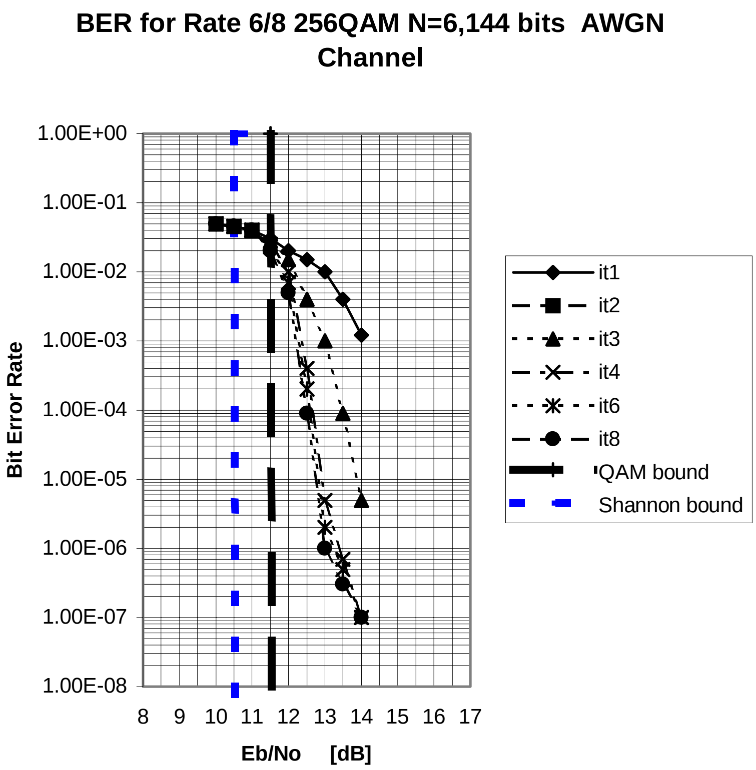

Figure 14

11. Coding And Modulation For 7 Bit/Hz Spectral Efficiency

This section investigated one scheme that use independent I and Q modulation. The scheme combines a rate 7/10 coding scheme with 1024QAM.

11.1 Puncturing

In order to obtain a rate 7/10 code, the puncturing pattern used is shown in Table 12.

Table 12. Puncturing and Mapping for Rate 7/10 1024QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}
parity bit (p)	p_1	-	-	-	-	p_6	-	-	-	-	p_{11}	-	-	-
parity bit (q)	-	-	q_3	-	-	-	-	q_8	-	-	-	-	q_{13}	-
32AM symbol (I)	$(d_1, d_2, d_3, p_1, q_3)$							$(d_8, d_9, d_{10}, d_{11}, q_8)$						
32AM symbol (Q)	$(d_4, d_5, d_6, d_7, p_6)$							$(d_{12}, d_{13}, d_{14}, p_{11}, q_{13})$						
1024QAM symbol	$(d_1,d_2,d_3,p_1,q_3,d_4,d_5,d_6,d_7,p_6)$							$(d_8,d_9,d_{10},d_{11},q_8,d_{12},d_{13},d_{14},p_{11},q_{13})$						

11.2 Modulation

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k\ u_9^k, u_{10}^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k, u_4^k and u_5^k modulate the I component and u_6^k, u_7^k, u_8^k, u_9^k and u_{10}^k modulate the Q component of a 1024QAM scheme. For a 1024QAM constellation with points at -31A, -29A, -27A, -25A, -23A, -21A, -19A, -17A, -15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A, 17A, 19A, 21A, 23A, 25A, 27A, 29A, 31A. E_{av} is:

$$E_{av} = 682\ A^2 \qquad (44)$$

For a rate 7/10 code and 512QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta\, E_b}{N_0}\right)^{-1} = 682\, A^2\left(\frac{2\ x\ 7\ x\, E_b}{N_0}\right)^{-1} = 48.7\, A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (45)$$

The puncturing and mapping scheme shown in Table 12 is for 14 consecutive information bits that are coded into 20 coded bits, therefore, two 1024QAM symbols. The turbo encoder is a rate 7/10 turbo code, which in conjunction with 1024QAM, gives a spectral efficiency of 7 bits/s/Hz.

11.3 Bit Probabilities

The 32AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k)$, where u_1^k is the most significant bit and u_5^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = { A_0 , A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 , A_8 , A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15} }
2. bit-1-is-1 = { A_{16} ,A_{17} , A_{18} , A_{19} , A_{20} , A_{21} , A_{22} , A_{23} , A_{24} ,A_{25} , A_{26} , A_{27} , A_{28} , A_{29} , A_{30} , A_{31} }
3. bit-2-is-0 = { A_0 , A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 , A_{16} ,A_{17} , A_{18} , A_{19} , A_{20} , A_{21} , A_{22} , A_{23} }
4. bit-2-is-1 = { A_8 , A_9 , A_{10} , A_{11} , A_{12} , A_{13} , A_{14} , A_{15}, A_{24} ,A_{25} , A_{26} , A_{27} , A_{28} , A_{29} , A_{30} , A_{31} }
5. bit-3-is-0 = { A_0 , A_1 , A_2 , A_3, A_8 , A_9 , A_{10} , A_{11}, A_{16} ,A_{17} , A_{18} , A_{19}, A_{24} ,A_{25} , A_{26} , A_{27} }
6. bit-3-is-1 = { A_4 , A_5 , A_6 , A_7, A_{12} , A_{13} , A_{14} , A_{15}, A_{20} , A_{21} , A_{22} , A_{23}, A_{28} , A_{29} , A_{30} , A_{31} }
7. bit-4-is-0 = { A_0 , A_1 , A_4 , A_5 , A_8 , A_9 , A_{12} , A_{13} , A_{16} ,A_{17} , A_{20} , A_{21} , A_{24} , A_{25} , A_{28} , A_{29} }
8. bit-4-is-1 = { A_2 , A_3 , A_6 , A_7 , A_{10} , A_{11} , A_{14} , A_{15} , A_{18} ,A_{19} , A_{22} , A_{23} , A_{26} , A_{27} , A_{30} , A_{31} }
9. bit-5-is-0 = { A_0 , A_2 , A_4 , A_6 , A_8 , A_{10} , A_{12} , A_{14} , A_{16} ,A_{18} , A_{20} , A_{22} , A_{24} , A_{26} , A_{28} , A_{30} }
10. bit-5-is-1 = { A_1 , A_3 , A_5 , A_7 , A_9 , A_{11} , A_{13} , A_{15} , A_{17} ,A_{19} , A_{21} , A_{23} , A_{25} , A_{27} , A_{29} , A_{31} }

From each received symbol, R^k, the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}1\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}1\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \quad (46)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}2\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}2\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \quad (47)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}3\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}3\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \quad (48)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}4\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}4\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \quad (49)$$

$$LLR(u_5^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}5\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_i \right\|\right)}{\sum_{A_j \in bit\text{-}5\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\left\| R^k - A_j \right\|\right)}\right) \quad (50)$$

11.4 Simulation Results

Figure 15 shows the simulation results for 2,048 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 17 dB.

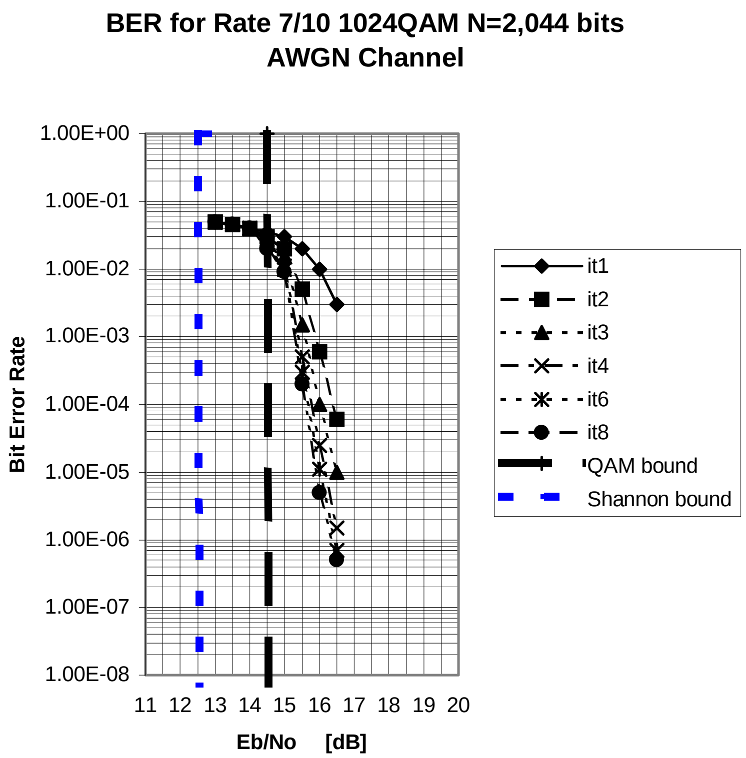

Figure 15

12. Power Vs. Bandwidth In An AWGN Channel

This section gives an estimate of the trade off which can be achieved between minimum required E_b/N_0 and bandwidth efficiency. An information data rate of 2,048 Mbit/s and a maximum transmitter delay of 1 ms is considered. The corresponding interleaver size is 2,048 bits.

11.1 Channel model

All the simulations assumed the additive white Gaussian noise (AWGN) channel model, with independent I and Q signals.

11.2 Simulation Results

Simulations were run for bandwidth efficiencies from 1 to 7 bit/symbol using the recommended coding and modulation schemes. The results are shown in Figures 16 to 22.

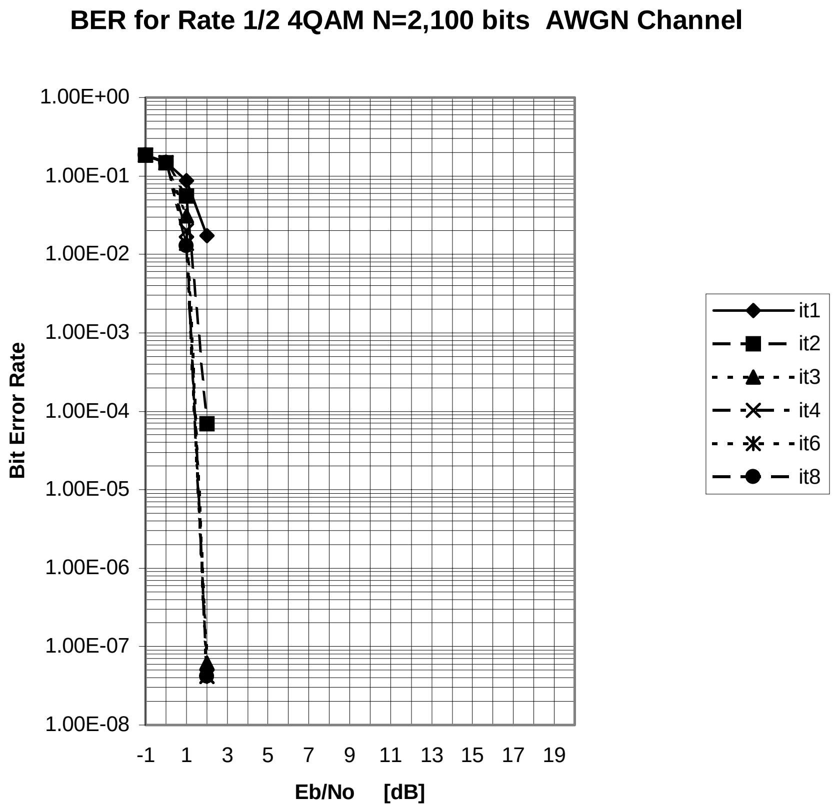

Figure 16

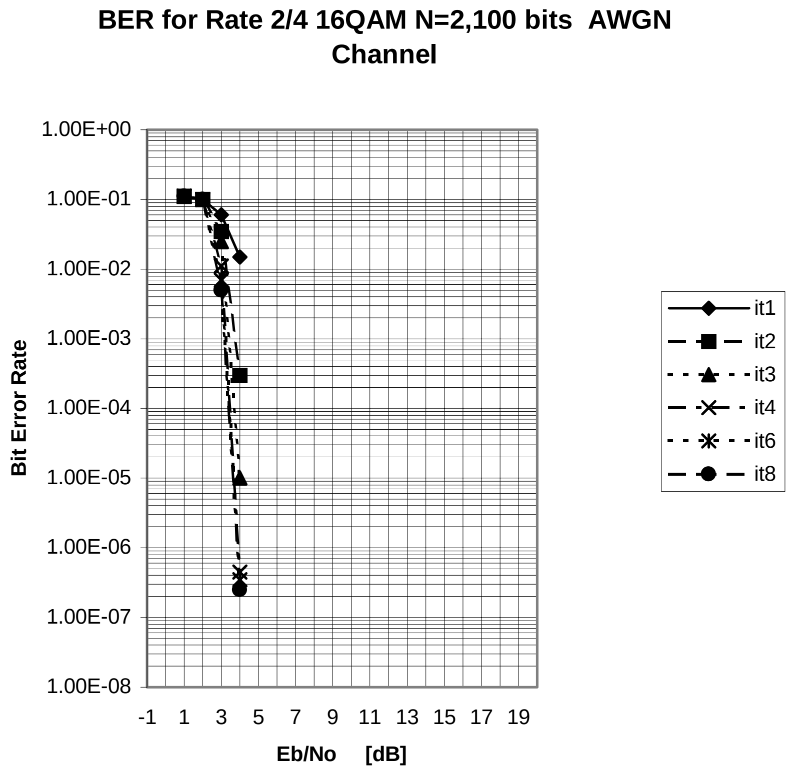

Figure 17

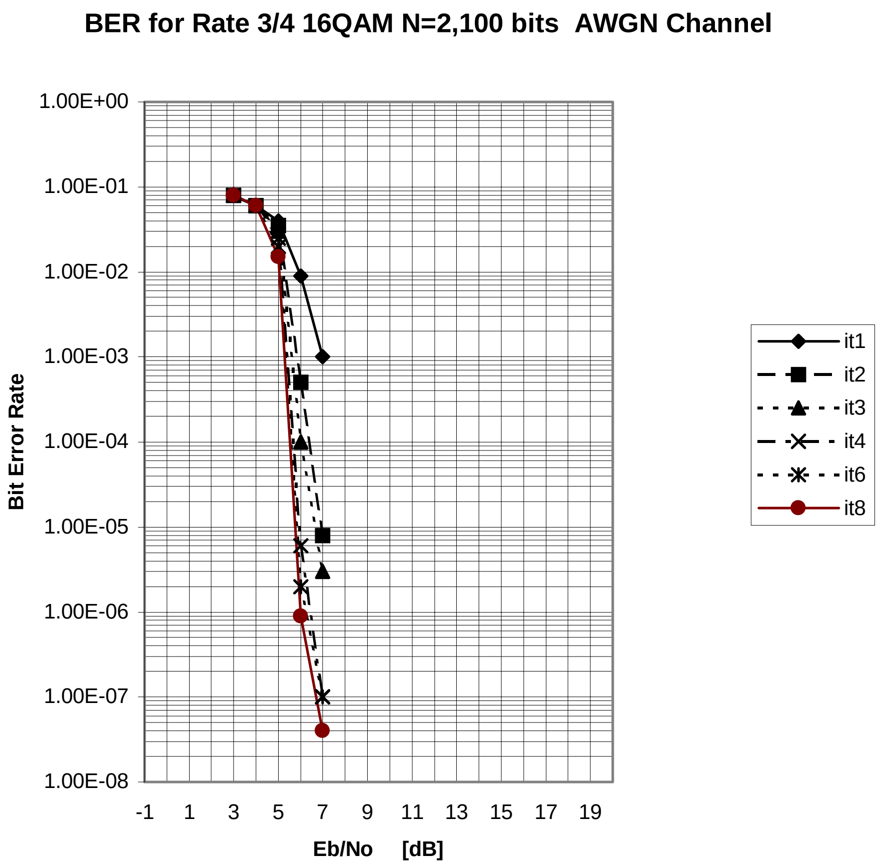

Figure 18

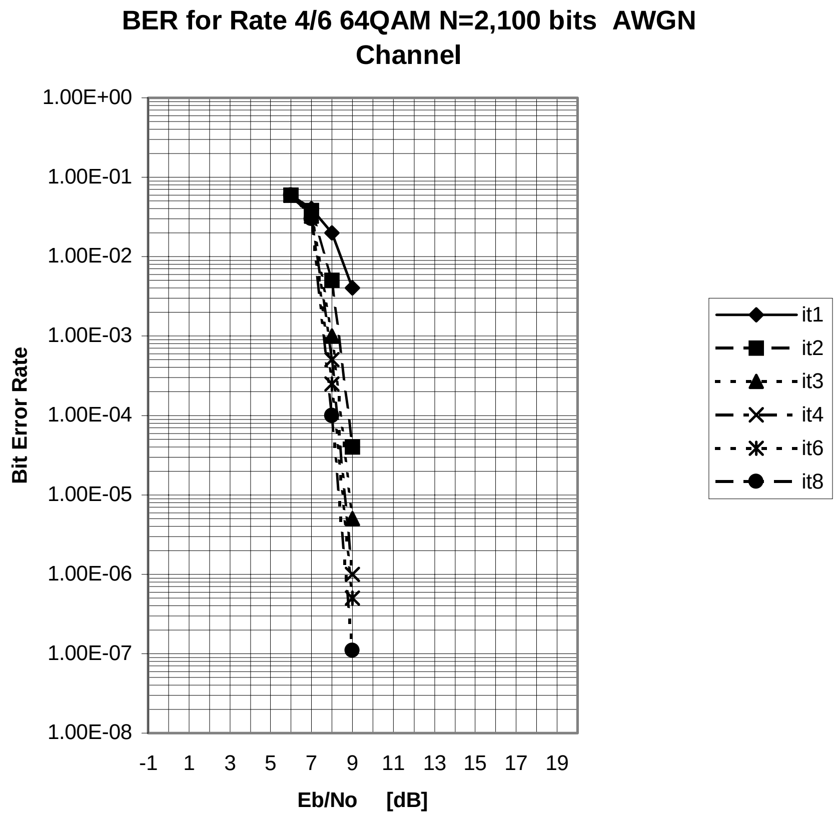

Figure 19

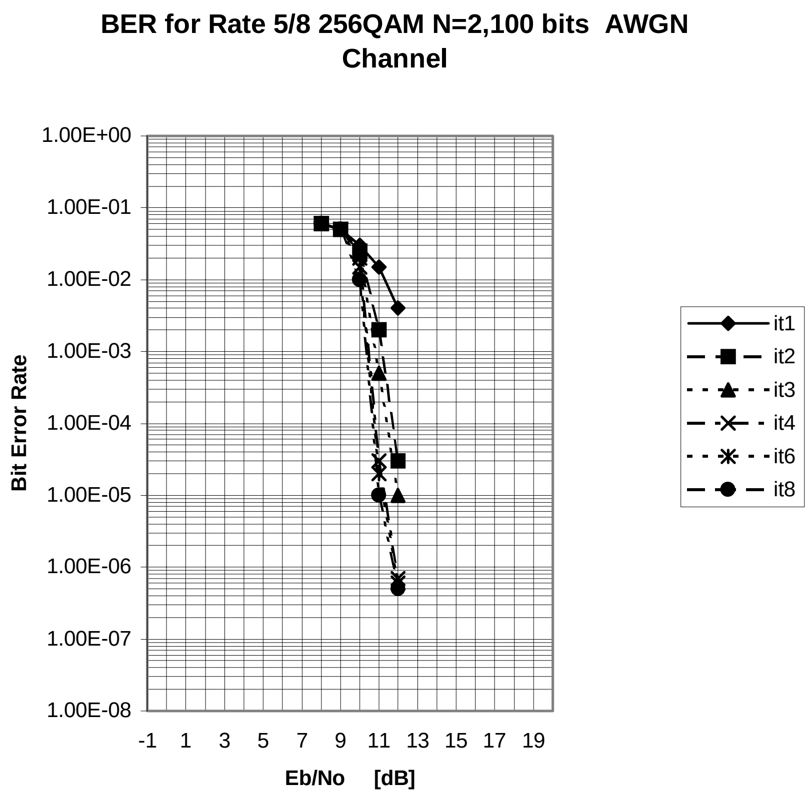

Figure 20

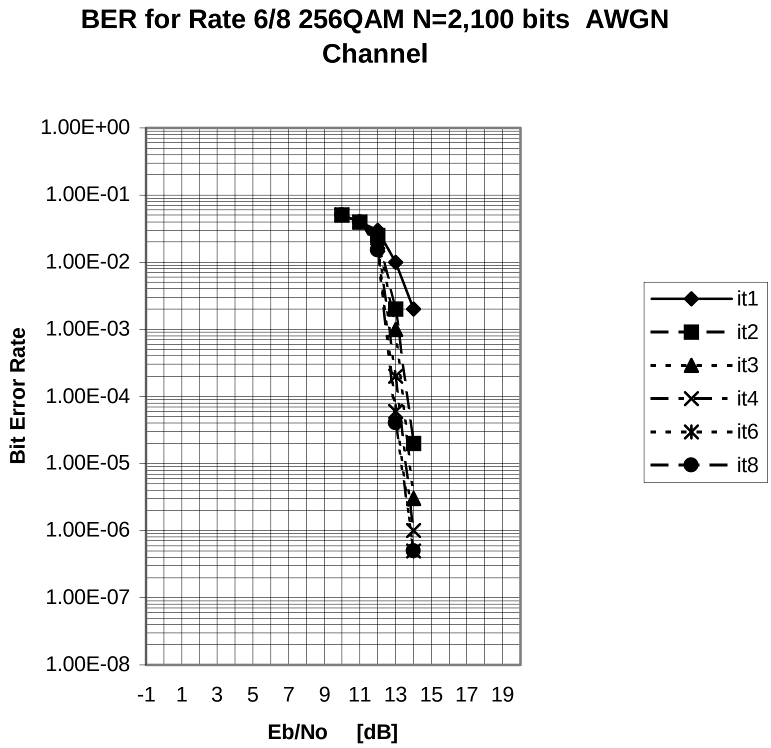

Figure 21

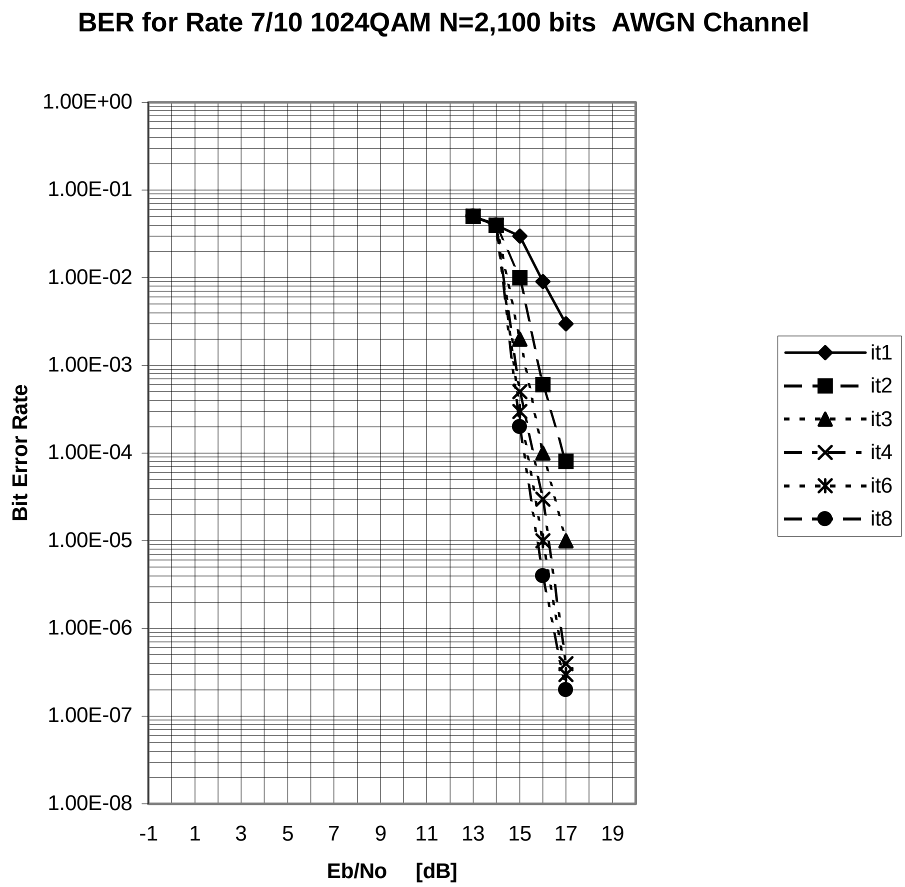

Figure 22

12.3 Conclusions

Table 13 summarizes the minimum E_b/N_0 required to achieve a BER of 10^{-7}.

Table 13 Minimum E_b/N_0 required to achieve a BER of 10^{-7}

Spectral efficiency η [bits/s/Hz]	Coding Rate And Modulation	Symbol Rate [ksym/s]	E_b/N_0 For BER = 10^{-7} [dB]
1	1/2 and 4QAM	2048	2.2
2	2/4 and 16QAM	1024	4.2
3	3/4 and 16QAM	682	6.5
4	4/6 and 64QAM	512	9.1
5	5/8 and 256QAM	408	12.3
6	6/8 and 256QAM	342	14.5
7	7/10 and 1024QAM	292	17.0

The results show the potential reduction in bandwidth for a given signal-to-noise ration for a particular channel.

13. Conclusions

Table 14 summarizes all simulation results from this study.

Table 14. Summary of Some Simulation Results.

Spectral efficiency η[bits/s/Hz]	Coding Rate	Modulation	Interleaver size in Information bits	Required E_b/N_0[dB] BER= 10^{-7}	QAM bound [dB]
1	1/2	4QAM	1,024	2.1	1.0
2	2/4	16QAM	256	6.8	2.1
2	2/4	16QAM	512	5.3	2.1
2	2/4	16QAM	768	4.9	2.1
2	2/4	16QAM	1,024	4.5	2.1
2	2/4	16QAM	2,048	4.2	2.1
2	2/4	16QAM	32,728	2.9	2.1
3	3/4	16QAM	2,048	6.5	4.6
3	3/4	16QAM	4,096	5.8	4.6
3	3/6	64QAM	4,096	6.1	4.3
4	4/6	64QAM	2,048	9.1	6.6
4	4/6	64QAM-1	4,096	8.3	6.6
4	4/6	64QAM-2	4,096	10.5	6.6
5	5/6	64QAM	5,120	13.0	9.0
5	5/8	256QAM	2,048	12.3	9.0
5	5/8	256QAM	5,120	11.8	9.0
6	6/8	256QAM	2,048	14.5	11.7
6	6/8	256QAM	6,144	14.2	11.7
7	7/10	1024QAM	2,044	17.0	14.5

14. Summary

This paper address point 4 of G.992.1.bis issues list and point 1.4 of G.992.2.bis issues list.

We propose that G.992.1.bis and G.992.2.bis support the Turbo codes describes in this document for Forward Error Correction. These results are so close to the QAM bounds that the Reed-Solomon is not needed. Instead of using Reed-Solomon as outer encoder we propose to use a longer interleaver in the turbo coder, that allows better results with the same global delay.