Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Method for using Concatenated Convolutional Turbo Codes in IEEE 802.16a**
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Re:	802.16aD4 and 802.16cD1
Abstract	This paper proposes a method for using concatenated convolutional Turbo codes for 802.16a. These concatenated convolutional turbo codes are optimized for each constellation size.
Purpose	To be used by TGa and TGc for discussion and to help preparing the draft document.
Notice	
Release	
Patent Policy and Procedures	

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Proposal

This paper proposes a method for using concatenated convolutional Turbo codes for 802.16a. These concatenated convolutional turbo codes are optimized for each constellation size.

Discussion

1. Introduction:

Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Since its discovery in 1993, a lot of research has been done in the application of turbo codes in deep space communications, mobile satellite/cellular communications, microwave links, paging, in OFDM and CDMA architectures. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel. The extra coding gain offered by turbo codes can be used either to minimize bandwidth or to reduce power requirements in the link budget. Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:

- Inmarsat's new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
- The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification. The IMT-2000 represents the third generation mobile radio systems worldwide standard. The 3GPP objective is to harmonize similar standards proposals from Europe, Japan, Korea and the United States.
- NASA's next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
- The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
- The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.

The way to avoid these computing requirements is to treat the QAM signal like two AM modulations (one in the I direction and one in the Q direction) and use the probabilities of the I and Q AM values as an input to the turbo code process.

Technique proposed here provides the most protected bits for the information bits and the least protected bits for the parity bits puncturing in applied to the parity bits.

2. Capacity Bounds

The minimum E_b/N_0 values to achieve the Shannon bound, 4QAM, 8QAM, 16QAM, 32QAM, 64QAM, 128QAM, 256QAM, 512QAM and 1024QAM bounds for spectral efficiencies from 2/3 up to 7 bits/s/Hz respectively are as in Table 1 for a BER=10^{-5}.

Table 1. Shannon and QAM bounds.

Spectral efficiency η [bit/s/Hz]	**Shannon bound [dB]**	**4 QAM bound [dB]**	**16 QAM bound [dB]**	**64 QAM bound [dB]**	**256 QAM bound [dB]**	**1024 QAM bound [dB]**
2/3	-0.5	0.3	-0.4	-0.4	-0.4	-0.4
1	0	1.0	0.1	0.1	0.1	0.1
2	1.75	∞	2.1	2.09	2.09	2.09
3	3.7	-	4.6	4.3	4.3	4.3
4	5.6	-	∞	6.6	6.6	6.6
5	7.9	-	-	9.1	9.0	9.0
6	10.3	-	-	∞	11.7	11.7
7	12.6	-	-	-	14.5	14.5

The conversion from E_s / N_0 to E_b / N_0 is performed using the following relation

$$E_b / N_0[dB] = E_s / N_0\,[dB] - 10 \log_{10}(\eta)\ \ [dB] \qquad (1)$$

where η is the number of information bits per symbol.

The required C/N_0 given a certain E_b/N_0 can be found using the following relation:

$$C/N_0\ [dB\text{-}Hz] = E_b/N_0\ [dB] + 10 \log_{10}(R_b)\ \ [dB\text{-}Hz] \qquad (2)$$

where R_b is the information bit rate.

For a D-dimension modulation the following formulae are used:

$$SNR = \frac{E\,[|\,a_k^2\,|]}{E\,[|\,w_k^2\,|]} = \frac{E\,[|\,a_k^2\,|]}{D\sigma_N^2} = \frac{E_{av}}{D\sigma_N^2} \qquad (3)$$

$$SNR = \frac{E_s}{D\frac{N_0}{2}} = \frac{\eta E_b}{D\frac{N_0}{2}} \qquad (4)$$

where σ^2_N is the noise variance in each of the D dimension and η is the number of information bits per symbol. From the above relations:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} \qquad (5)$$

3. Coding

The proposed coding scheme is shown in Figure 1.The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

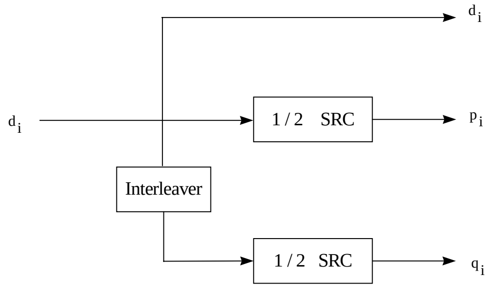

Figure 1

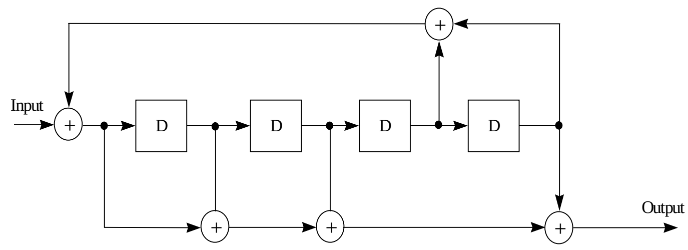

Figure 2

4 Turbo code internal interleaver design (3GPP).

The Turbo code internal interleaver consists of bits-input to a rectangular matrix, intra-row and inter-row permutations of the rectangular matrix, and bits-output from the rectangular matrix with pruning. The bits input to the Turbo code internal interleaver are denoted by $x_1, x_2, x_3, \ldots, x_K$, where K is the integer number of the bits and takes one value of $40 \le K \le 32000$. The relation between the bits input to the Turbo code internal interleaver and the bits input to the channel coding is defined by $x_k = o_{irk}$ and $K = K_i$.

K Number of bits input to Turbo code internal interleaver
R Number of rows of rectangular matrix
C Number of columns of rectangular matrix
p Prime number
v Primitive root
s(i) Base sequence for intra-row permutation
qj Minimum prime integers
rj Permuted prime integers
T(j) Inter-row permutation pattern
Uj(i) Intra-row permutation pattern
i Index of matrix
j Index of matrix
k Index of bit sequence

4.1 Bits-input to rectangular matrix

The bit sequence input to the Turbo code internal interleaver x_k is written into the rectangular matrix as follows.

(1) Determine the number of rows R of the rectangular matrix such that:

$$R = \begin{cases} 5, \text{if } (40 \le K \le 159) \\ 10, \text{if } ((160 \le K \le 200) \text{ or } (481 \le K \le 530)) \\ 20, \text{if } (K = \text{any other value}) \end{cases}$$

where the rows of rectangular matrix are numbered 0, 1, 2, …, R - 1 from top to bottom.

(2) Determine the number of columns C of rectangular matrix such that:

if (481 $\le K \le$ 530) then

$p = 53$ and $C = p$.

else
Find minimum prime p such that

$(p + 1) - K/R \ge 0$,

and determine C such that

if ($p - K/R \ge 0$) then

if ($p - 1 - K/R \ge 0$) then

$C = p - 1$.
else

$C = p$.

end if

else

$C = p + 1$

end if

end if

where the columns of rectangular matrix are numbered 0, 1, 2, …, C - 1 from left to right.

(3) Write the input bit sequence x_k into the $R \times C$ rectangular matrix row by row starting with bit x_1 in column
0 of row 0:

$$\begin{bmatrix} x_1 & x_2 & x_3 & \cdots\ x_C \\ x_{(C+1)} & x_{(C+2)} & x_{(C+3)} & \cdots x_{2C} \\ \vdots & \vdots & \vdots & \ldots\ \vdots \\ x_{((R-1)C+1)} & x_{((R-1)C+2)} & x_{((R-1)C+3)} & \cdots x_{RC} \end{bmatrix}$$

4.2 Intra-row and inter-row permutations

After the bits-input to the $R \times C$ rectangular matrix, the intra-row and inter-row permutations for the $R \times C$ rectangular matrix are performed by using the following algorithm.

(1) Select a primitive root v from table 2.

(2) Construct the base sequence $s(i)$ for intra-row permutation as:

$s(i) = [v \times s(i - 1)] \bmod p$, $i = 1, 2, \ldots, (p - 2)$., and $s(0) = 1$.

(3) Let $q_0 = 1$ be the first prime integer in $\{q_j\}$, and select the consecutive minimum prime integers $\{q_j\}$ ($j = 1, 2, \ldots, R - 1$) such that:

g.c.d$\{q_j, p - 1\} = 1$, $q_j > 6$, and $q_j > q_{(j-1)}$,

where g.c.d. is greatest common divisor.

(4) Permute $\{q_j\}$ to make $\{r_j\}$ such that

$r_{T(j)} = q_j$, $j = 0, 1, \ldots, R - 1$,

where $T(j)$ ($j = 0, 1, 2, \ldots, R - 1$) is the inter-row permutation pattern defined as the one of the following four kind of patterns: Pat_1, Pat_2, Pat_3 and Pat_4 depending on the number of input bits K.

$$\{T(0), T(1), T(2), \ldots, T(R-1)\} = \begin{cases} Pat_4 & \text{if } (40 \le K \le 159) \\ Pat_3 & \text{if } (160 \le K \le 200) \\ Pat_1 & \text{if } (201 \le K \le 480) \\ Pat_3 & \text{if } (481 \le K \le 530) \\ Pat_1 & \text{if } (531 \le K \le 2280) \\ Pat_2 & \text{if } (2281 \le K \le 2480) \\ Pat_1 & \text{if } (2481 \le K \le 3160) \\ Pat_2 & \text{if } (3161 \le K \le 3210) \\ Pat_1 & \text{if } (3211 \le K) \end{cases},$$

where Pat_1, Pat_2, Pat_3 and Pat_4 have the following patterns respectively.

Pat_1: {19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 10, 8, 13, 17, 3, 1, 16, 6, 15, 11}

Pat_2: {19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 16, 13, 17, 15, 3, 1, 6, 11, 8, 10}

Pat_3: {9, 8, 7, 6, 5, 4, 3, 2, 1, 0}

Pat_4: {4, 3, 2, 1, 0}

(5) Perform the j-th ($j = 0, 1, 2, \ldots, R - 1$) intra-row permutation as:

if ($C = p$) then

$U_j(i) = s([i\times r_j]\text{mod}(p - 1))$, $i = 0, 1, 2, \ldots, (p - 2)$.,   and $U_j(p - 1) = 0$,

where $U_j(i)$ is the input bit position of *i*-th output after the permutation of *j*-th row.

end if

if ($C = p + 1$) then

$U_j(i) = s([i\times r_j]\text{mod}(p - 1))$, $i = 0, 1, 2, \ldots, (p - 2)$., $U_j(p - 1) = 0$, and $U_j(p) = p$,

where $U_j(i)$ is the input bit position of *i*-th output after the permutation of *j*-th row, and

if ($K = C \times R$) then

Exchange $U_{R-1}(p)$ with $U_{R-1}(0)$.

end

if

end if

if ($C = p - 1$) then

$U_j(i) = s([i\times r_j]\text{mod}(p - 1)) - 1$, $i = 0, 1, 2, \ldots, (p - 2)$,

where $U_j(i)$ is the input bit position of *i*-th output after the permutation of *j*-th row.
end if

(6) Perform the inter-row permutation based on the pattern $T(j)$ ($j = 0, 1, 2, \ldots, R - 1$),
where $T(j)$ is the original row position of the *j*-th permuted row.

Table 2: Table of prime *p* and associated primitive root *v*

p	v	p	v	p	v	p	v	p	v
7	3	313	10	709	2	1129	11	1597	11
11	2	317	2	719	11	1151	17	1601	3
13	2	331	3	727	5	1153	5	1607	5
17	3	337	10	733	6	1163	5	1609	7
19	2	347	2	739	3	1171	2	1613	3
23	5	349	2	743	5	1181	7	1619	2
29	2	353	3	751	3	1187	2	1621	2
31	3	359	7	757	2	1193	3	1627	3
37	2	367	6	761	6	1201	11	1637	2
41	6	373	2	769	11	1213	2	1657	11
43	3	379	2	773	2	1217	3	1663	3
47	5	383	5	787	2	1223	5	1667	2
53	2	389	2	797	2	1229	2	1669	2
59	2	397	5	809	3	1231	3	1693	2
61	2	401	3	811	3	1237	2	1697	3
67	2	409	21	821	2	1249	7	1699	3
71	7	419	2	823	3	1259	2	1709	3
73	5	421	2	827	2	1277	2	1721	3
79	3	431	7	829	2	1279	3	1723	3
83	2	433	5	839	11	1283	2	1733	2
89	3	439	15	853	2	1289	6	1741	2
97	5	443	2	857	3	1291	2	1747	2
101	2	449	3	859	2	1297	10	1753	7
103	5	457	13	863	5	1301	2	1759	6
107	2	461	2	877	2	1303	6	1777	5
109	6	463	3	881	3	1307	2	1783	10
113	3	467	2	883	2	1319	13	1787	2
127	3	479	13	887	5	1321	13	1789	6
131	2	487	3	907	2	1327	3	1801	11
137	3	491	2	911	17	1361	3	1811	6
139	2	499	7	919	7	1367	5	1823	5
149	2	503	5	929	3	1373	2	1831	3
151	6	509	2	937	5	1381	2	1847	5
157	5	521	3	941	2	1399	13	1861	2
163	2	523	2	947	2	1409	3	1867	2
167	5	541	2	953	3	1423	3	1871	14
173	2	547	2	967	5	1427	2	1873	10
179	2	557	2	971	6	1429	6	1877	2
181	2	563	2	977	3	1433	3	1879	6
191	19	569	3	983	5	1439	7	1889	3
193	5	571	3	991	6	1447	3	1901	2
197	2	577	5	997	7	1451	2	1907	2
199	3	587	2	1009	11	1453	2	1913	3
211	2	593	3	1013	3	1459	3	1931	2
223	3	599	7	1019	2	1471	6	1933	5
227	2	601	7	1021	10	1481	3	1949	2
229	6	607	3	1031	14	1483	2	1951	3
233	3	613	2	1033	5	1487	5	1973	2
239	7	617	3	1039	3	1489	14	1979	2
241	7	619	2	1049	3	1493	2	1987	2
251	6	631	3	1051	7	1499	2	1993	5
257	3	641	3	1061	2	1511	11	1997	2
263	5	643	11	1063	3	1523	2	1999	3
269	2	647	5	1069	6	1531	2		

p	v	p	v	p	v	p	v	p	v
271	6	653	2	1087	3	1543	5		
277	5	659	2	1091	2	1549	2		
281	3	661	2	1093	5	1553	3		
283	3	673	5	1097	3	1559	19		
293	2	677	2	1103	5	1567	3		
307	5	683	5	1109	2	1571	2		
311	17	691	3	1117	2	1579	3		

4.3 Bits-output from rectangular matrix with pruning

After intra-row and inter-row permutations, the bits of the permuted rectangular matrix are denoted by y'_k:

$$\begin{bmatrix} y'_1 & y'_{(R+1)} & y'_{(2R+1)} & \cdots y'_{((C-1)R+1)} \\ y'_2 & y'_{(R+2)} & y'_{(2R+2)} & \cdots y'_{((C-1)R+2)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y'_R & y'_{2R} & y'_{3R} & \cdots & y'_{CR} \end{bmatrix}$$

The output of the Turbo code internal interleaver is the bit sequence read out column by column from the intra-row and inter-row permuted $R \times C$ matrix starting with bit y'_1 in row 0 of column 0 and ending with bit y'_{CR} in row R - 1 of column C - 1. The output is pruned by deleting bits that were not present in the input bit sequence, i.e. bits y'_k that corresponds to bits x_k with $k > K$ are removed from the output. The bits output from Turbo code internal interleaver are denoted by $x'_1, x'_2, \ldots, x'_K$, where x'_1 corresponds to the bit y'_k with smallest index k after pruning, x'_2 to the bit y'_k with second smallest index k after pruning, and so on. The number of bits output from Turbo code internal interleaver is K and the total number of pruned bits is: $R \times C - K$.

5. Modulation For 1 Bit/s/Hz Spectral Efficiency.

The scheme proposed in this case combines a rate 1/2 coding scheme with 4QAM.

5.1 Puncturing

In order to obtain a code rate of 1/2, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 3.

Table 3. Puncturing and Mapping for Rate 1/2 4QAM.

Information bit (d)	d_1	d_2
Parity bit (p)	p_1	-
Parity bit (q)	-	q_2
2AM symbol (I)	$(u_1) = (d_1)$	$(u_1) = (d_2)$
2AM symbol (Q)	$(u_2) = (p_1)$	$(u_2) = (q_2)$
4QAM symbol (I , Q)	$(I,Q) = (u_1, u_2) = (d_1, p_1)$	$(I,Q) = (u_1, u_2) = (d_2, q_2)$

5.2.Modulation

A 4QAM scheme is shown in Figure 3. An equivalent 2AM modulation is shown in Figure 4.

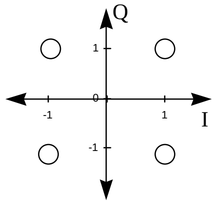

Figure 3

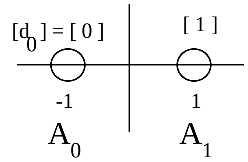

Figure 4

At time k, the symbol $u^k = (u_1^k, u_2^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 4QAM constellation with points at -A and A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2)}{4} = 2A^2 \qquad (6)$$

For a rate 1/24 code and 4QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right) = 2A^2\left(\frac{2 \, x \, 1 \, x \, E_b}{N_0}\right)^{-1} = A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (7)$$

5.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{1} \exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{1} \exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2]}\right) \qquad (8)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{1} \exp[-\frac{1}{2\sigma_N^2}\left(Q^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{1} \exp[-\frac{1}{2\sigma_N^2}\left(Q^k - a_{0,i}^k\right)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(Q^k - B_1\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(Q^k - B_0\right)^2]}\right) \qquad (9)$$

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 4 LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 2 bit-LLR values are reduced to one term each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non-punctured scheme.

5.4 Simulations Results.

Figure 5 shows the performance of a turbo code using a 1024 bit S-type interleaver. The target BER of 10^{-7} for a 1,024 information bit interleaver can be achieve at $E_b/N_0 = 2.1$ dB

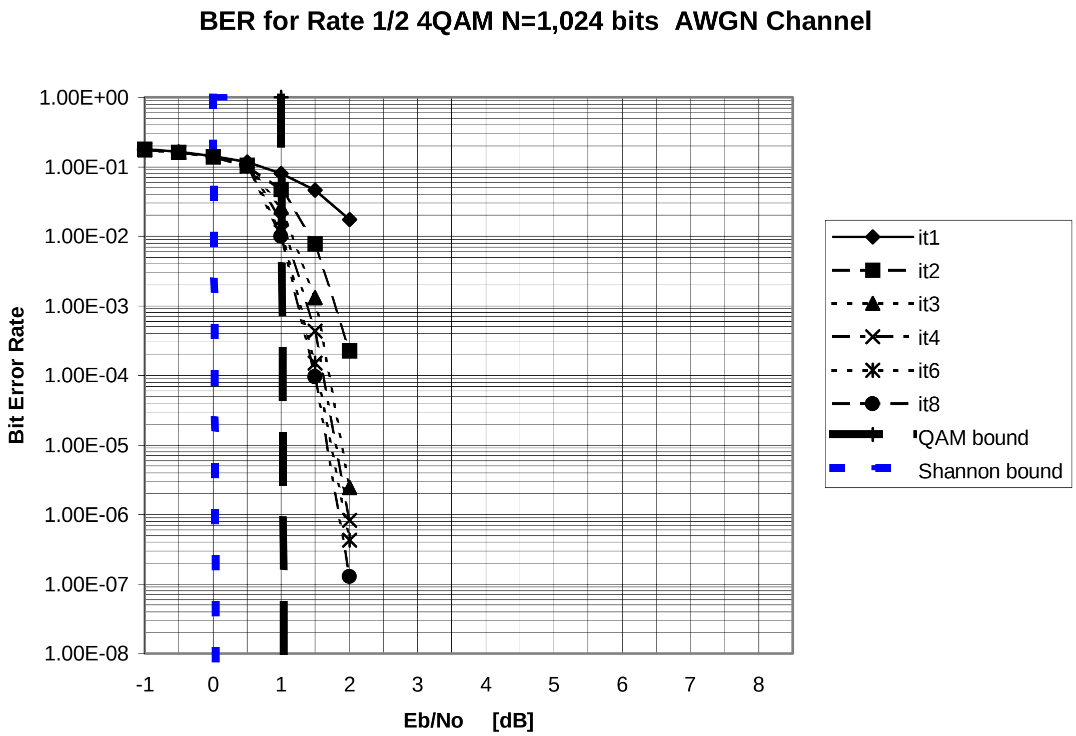

Figure 5

6. Coding And Modulation For 2 Bit/s/Hz Spectral Efficiency.

The investigated combines 2/4 coding scheme with 16QAM.

6.1 Puncturing.

In order to obtain a rate 2/4 code, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 4.

Table 4. Puncturing and Mapping for Rate 2/4 16QAM.

Information bit (d)	d_1	d_2
parity bit (p)	p_1	-
parity bit (q)	-	q_2
4AM symbol (I)	$(u_1, u_2) = (d_1, p_1)$	
4AM symbol (Q)	$(u_3, u_4) = (d_2, p_2)$	
16 QAM symbol (I , Q)	$(I, Q) = (u_1, u_2, u_3, u_4) = (d_1, p_1, d_2, p_2)$	

6.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$ is sent through the channel and the point r^k in two dimensional space is received. For a 16QAM constellation with points at -3A, -A,A and 3A, The E_{av} is:

$$E_{av} = \frac{4(A^2 + A^2) + 8(A^2 + 9A^2) + 4(9A^2 + 9A^2)}{16} = 10A^2 \qquad (10)$$

For a rate 1/2 code and 16QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right) = 10\,A^2\left(\frac{2\,x\,2\,x\,E_b}{N_0}\right)^{-1} = 2.5\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (11)$$

It is assumed that the time k, u_1^k and u_2^k modulate the I component and u_3^k and u_4^k modulate the Q component for a 16QAM scheme. The symbol u^k symbol has the following mapping: $u^k = (u_1^k, u_2^k, u_3^k, u_4^k) = (d^i, p^i, d^{i+1}, q^{i+1})$. The parity bits are mapped to the least protected bits of the QAM symbol. Note that k denotes the symbol time index and i the information bit time index. This means a puncturing of one in two parity bits. Considering two independent Gaussian noise sources with identical variance σ^2_N, the LLR can be determined independently for each I and Q.

At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the 4 bit-LLR values. The mapping of the information bit is made to the most protected bit in each dimension (u_1^k for the I signal and u_3^k for the Q signal). In order to estimate the performance of this scheme, rate 1/2 turbo code and 4AM modulation is used, instead of 16QAM modulation.

6.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2]}\right) \quad (12)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2]}\right) \quad (13)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_2)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_1)^2]}\right) \quad (14)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{1,i}^k)^2]}{\sum_{i=1}^{2}\exp[-\frac{1}{2\sigma_N^2}(Q^k - a_{0,i}^k)^2]}\right) = \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_3)^2]}{\exp[-\frac{1}{2\sigma_N^2}(Q^k - B_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(Q^k - B_2)^2]}\right) \quad (15)$$

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 16LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 4 bit-LLR values are reduced to 2 terms each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non punctured scheme.

6.4 Simulations Results.

The rate 2/4 16QAM scheme described in this chapter achieves the target BER at less than 1.5 dB from capacity. The implementation in hardware is feasible and it can be used at very high data rates. The target BER of 10^{-7} can be achieved at E_b/N_0 = 4.5 dB for N=1024 information bits, as shown in Figure 6.

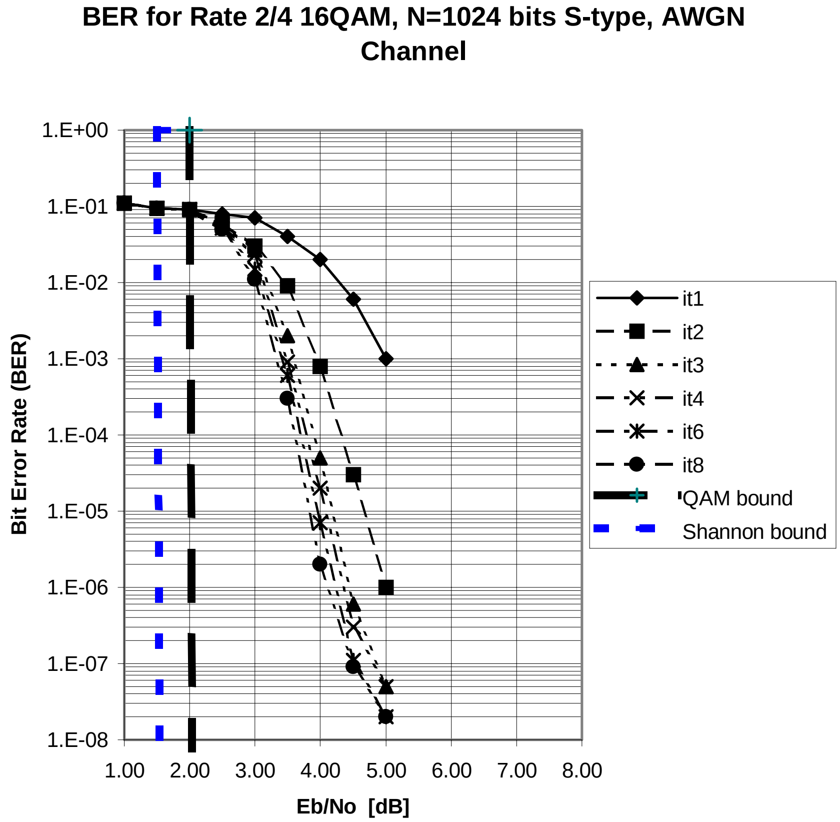

Figure 6

7. Coding And Modulation For 3 Bit/s/Hz Spectral Efficiency.

Two options are investigated in this section. The first scheme combines a rate 3/4 coding scheme with 16QAM. The second scheme combines a rate 3/4 coding scheme with 64QAM.

7.1 Option 1 Rate 3/4 Turbo code and 16QAM

7.1.1 Puncturing.

In order to obtain a rate 3/4 code, the puncturing pattern given in Table 5 is used.

Table 5. Puncturing and Mapping for Rate 3/4 16QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	-	p_2	-	-	-	-
parity bit (q)	-	-	-	-	q_5	-
4AM symbol (I)	(d_1, d_2)			(d_4, d_5)		
4AM symbol (Q)	(d_3, p_2)			(d_6, p_5)		
16 QAM symbol (I ,Q)	$(I, Q) = (d_1, d_2, d_3, p_2)$			$(I, Q) = (d_4, d_5, d_6, q_5)$		

7.1.2 Modulation

It is assumed that at time k, u_1^k and u_2^k modulate the I component and u_3^k and u_4^k modulate the Q component of a 16QAM scheme. In order to estimate the performance of this scheme, rate 3/4 turbo codes and 4 AM modulation are used. For a rate 3/4 code and 16QAM, noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 10\,A^2\left(\frac{2x3x\,E_b}{N_0}\right)^{-1} = \frac{10}{6}A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (16)$$

The puncturing and mapping scheme is shown in Table 5 for 6 consecutive information bits that are encoded into 8 coded bits, therefore two 16QAM symbols.

7.1.3 Bit Probabilities

For each received symbol, the bit probabilities are computed as described in equations (12)-(15).

7.1.4 Simulation Results

Figure 7 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 5.75 dB for N = 6144 information bits.

7.2 Option 2 Rate 3/6 Turbo code and 64QAM

7.2.1 Puncturing.

In order to obtain a rate 3/6 code, the puncturing pattern used is given in Table 6.

Table 6. Puncturing and Mapping for Rate 3/6 64QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	p_1	-	p_3	-	p_5	-
parity bit (q)	-	q_2	-	q_4	-	q_6
8AM symbol (I)	(d_1, d_2, p_1)			(d_4, d_5, q_4)		
8AM symbol (Q)	(d_3, p_3, q_2)			(d_6, p_5, q_6)		
64 QAM symbol (I , Q)	$(I, Q)=(d_1,d_2, p_1,d_3, p_3,q_2)$			$(I, Q)= (d_4,d_5, q_4,d_6, p_5,q_6)$		

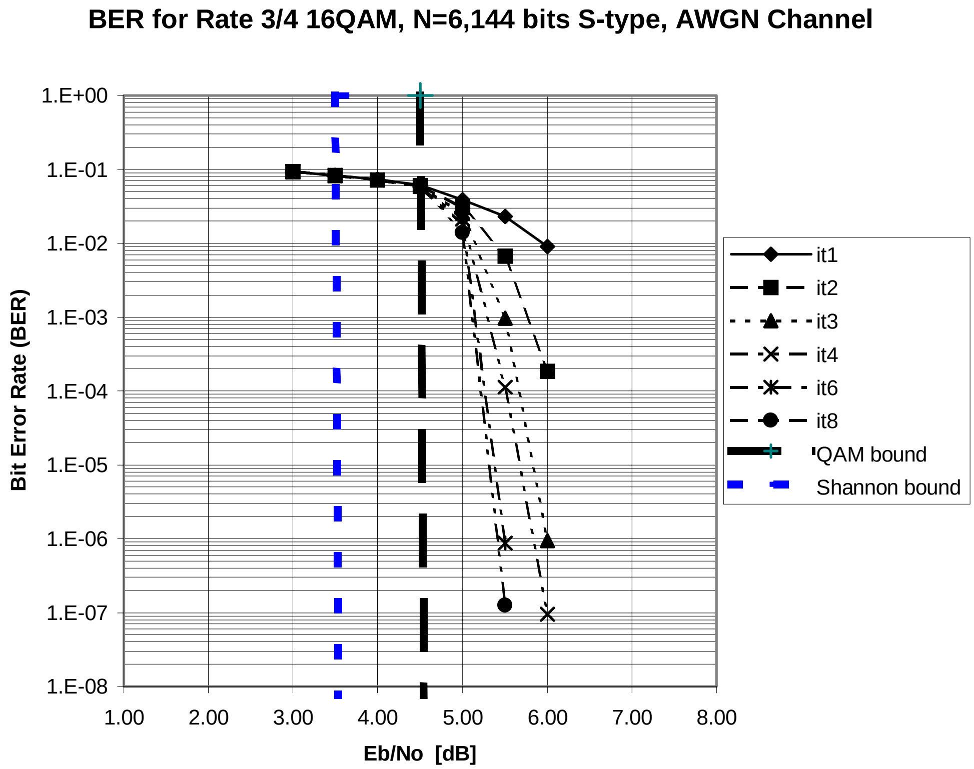

Figure 7

7.2.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k)$, is sent through the channel and the point r^k in two dimensional space is received. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of a 64QAM scheme. For 64QAM constellations with points at -7A, -5A, -3A, -A, A, 3A, 5A, 7A The E_{av} is:

$$E_{av} = (8(49+25+9+1)+8(25+49+49+9+49+1)+8(25+9+25+1)+8(9+1))\ A^2 / 64 = 42\ A^2 \quad (17)$$

For a rate 3/6 code and 64QAM, the noise variance in each dimension is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 42\,A^2\left(\frac{2x3x\,E_b}{N_0}\right)^{-1} = 7\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \quad (18)$$

In order to estimate the performance of this scheme, when rate 3/6 turbo codes and 8AM modulation is used. The puncturing and mapping scheme is shown in Table 6 for 6 consecutive information bits that are encoded into 12 coded bits, therefore two 64QAM symbols.

7.2.3 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for the I dimension:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) =$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2]}\right) \quad (19)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) =$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2]}\right) \quad (20)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{1,i}^k\right)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}\left(I^k - a_{0,i}^k\right)^2]}\right) =$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_1\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_5\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_3\right)^2] + \exp[-\frac{1}{2\sigma_n^2}\left(I^k - A_7\right)^2]}{\exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_0\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_4\right)^2] + \exp[-\frac{1}{2\sigma_N^2}\left(I^k - A_2\right)^2 + \exp[\frac{1}{2\sigma_N^2}\left(I^k - A_6\right)^2]}\right) \quad (21)$$

An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

7.2.4 Simulation Results

Figure 8 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 6.1 dB for N = 6,144 information bits. This result is 0.5 dB worse than the performance of the rate 3/4 16QAM scheme.

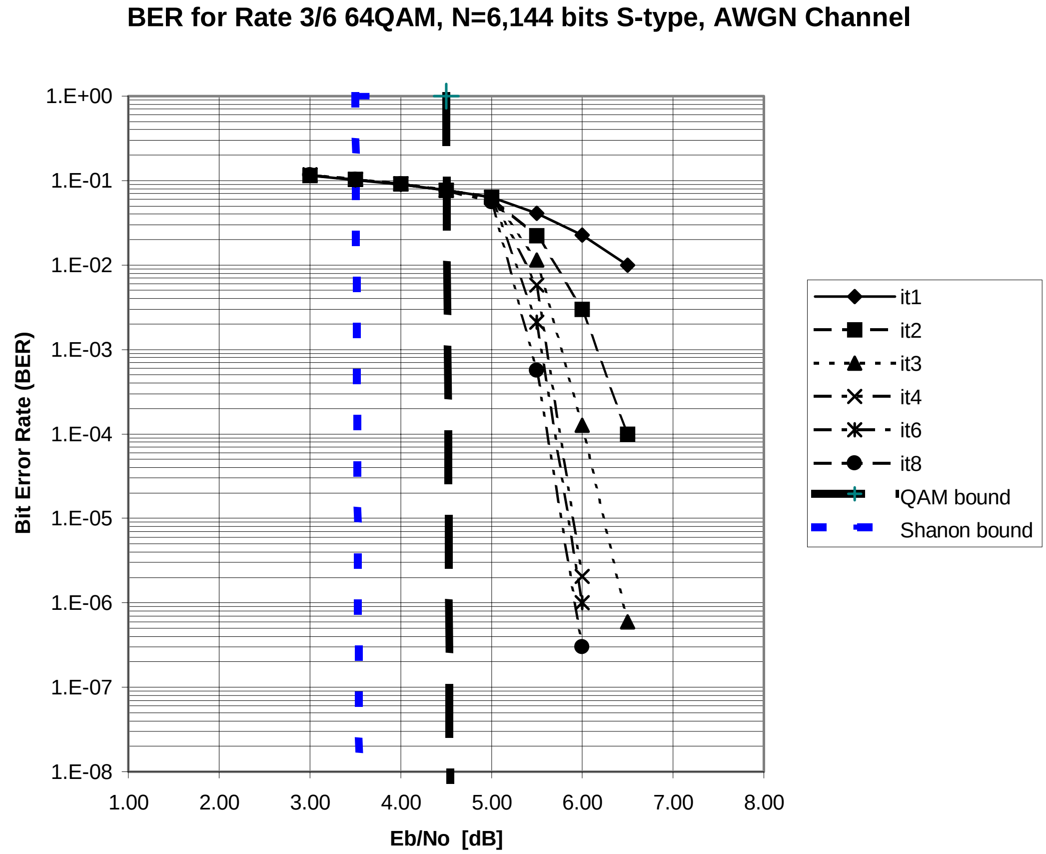

Figure 8

8. Coding And Modulation For 4 Bit/s/Hz Spectral Efficiency.

This scheme uses independent I and Q mapping and also uses Gray mapping in each dimension.

8.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing pattern used is shown in Table 7.

Table 7. Puncturing and Mapping for Rate 4/6 64QAM Option 1

Information bit (d)	d_1	d_2	d_3	d_4
parity bit (p)	p_1	-	-	-
parity bit (q)	-	-	q_3	-
8AM symbol (I)	(d_1, d_2, p_1)			
8AM symbol (Q)	(d_3, d_4, q_3)			
64 QAM symbol (I, Q)	$(I, Q)=(d_1, d_2, p_1, d_3, d_4, q_3)$			

8.2 Modulation

Gray mapping was used in each dimension. Four information bits are required to be sent using a 64QAM constellation. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 42\,A^2\left(\frac{2 \text{ x } 4 \text{ x } E_b}{N_0}\right)^{-1} = 5.25\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \qquad (22)$$

The puncturing and mapping scheme is shown in Table 7 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 7 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ^2_N, the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

8.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \qquad (23)$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_4)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_5)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_6)^2] + \exp[-\frac{1}{2\sigma_n^2}(I^k - A_7)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2 + \exp[\frac{1}{2\sigma_N^2}(I^k - A_3)^2]}\right)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \qquad (24)$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_6)^2] + \exp[-\frac{1}{2\sigma_n^2}(I^k - A_7)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_4)^2 + \exp[\frac{1}{2\sigma_N^2}(I^k - A_5)^2]}\right)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{1,i}^k)^2]}{\sum_{i=1}^{4}\exp[-\frac{1}{2\sigma_N^2}(I^k - a_{0,i}^k)^2]}\right) = \qquad (25)$$

$$= \log\left(\frac{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_1)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_5)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_3)^2] + \exp[-\frac{1}{2\sigma_n^2}(I^k - A_7)^2]}{\exp[-\frac{1}{2\sigma_N^2}(I^k - A_0)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_4)^2] + \exp[-\frac{1}{2\sigma_N^2}(I^k - A_2)^2 + \exp[\frac{1}{2\sigma_N^2}(I^k - A_6)^2]}\right)$$

For I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate $LLR(u_4^k)$, $LLR(u_5^k)$ and $LLR(u_6^k)$.

8.4 Simulation Results

Figure 9 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 8.3 dB.

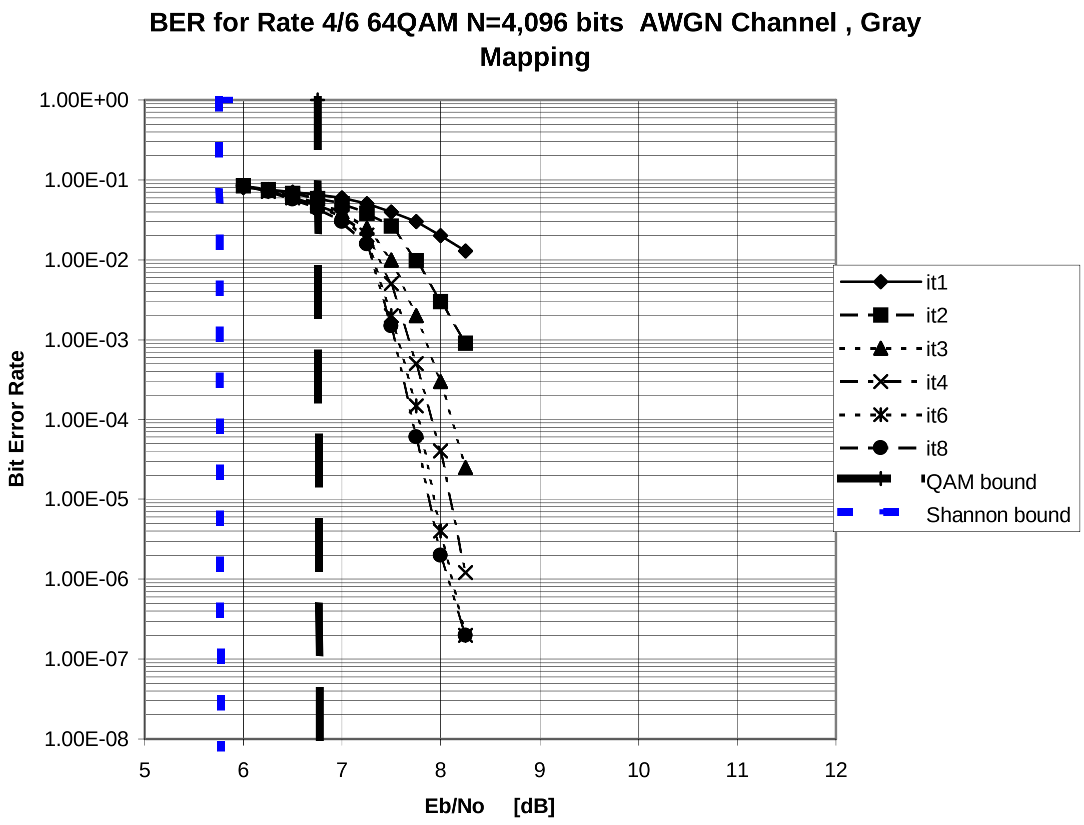

Figure 9.

9. Coding And Modulation For 5 Bit/s/Hz Spectral Efficiency.

This section investigated a rate 5/8 coding scheme with 256QAM.

9.1 Puncturing

In order to obtain a rate 5/8 code, the puncturing pattern used is shown in Table 8.

Table 8. Puncturing and Mapping for Rate 5/8 256QAM

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
parity bit (p)	p_1	-	-	-	p_5	-	-	p_8	-	-
parity bit (q)	-	-	q_3	-		q_6	-	-	-	q_{10}
16AM symbol (I)	(d_1, d_2, d_3, p_1)					(d_6, d_7, d_8, q_6)				
16AM symbol (Q)	(d_4, d_5, q_3, p_5)					$(d_9, d_{10}, p_8, q_{10})$				
256 QAM symbol (I , Q)	$(d_1, d_2, d_3, p_1, d_4, d_5, q_3, p_5)$					$(d_6, d_7, d_8, q_6, d_9, d_{10}, p_8, q_{10})$				

9.2 Modulation

For a 256QAM constellation with points at –15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A. E_{av} is:

$$E_{av} = 170\,A^2 \tag{26}$$

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k and u_4^k modulate the I component and u_5^k, u_6^k, u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 5/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 170\,A^2\left(\frac{2 \times 5 \times E_b}{N_0}\right)^{-1} = 17\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \tag{27}$$

In order to study the performance of this scheme, a rate 5/8 turbo code and a 16AM is used. The 256QAM scheme will achieve a similar performance in terms of bit error rate (BER) at twice the spectral efficiency, assuming an ideal demodulator. The puncturing and mapping scheme shown in Table 8 is for 10 consecutive information bits that are coded into 16 encoded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 5/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 5 bits/s/Hz.

9.3 Bit Probabilities

The 16AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit . The following set can be defined.

1. bit-1-is-0 = $\{ A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7 \}$
2. bit-1-is-1 = $\{ A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15} \}$
3. bit-2-is-0 = $\{ A_0, A_1, A_2, A_3, A_8, A_9, A_{10}, A_{11} \}$
4. bit-2-is-1 = $\{ A_4, A_5, A_6, A_7, A_{12}, A_{13}, A_{14}, A_{15} \}$
5. bit-3-is-0 = $\{ A_0, A_1, A_4, A_5, A_8, A_9, A_{12}, A_{13} \}$
6. bit-3-is-1 = $\{ A_2, A_3, A_6, A_7, A_{10}, A_{11}, A_{14}, A_{15} \}$
7. bit-4-is-0 = $\{ A_0, A_2, A_4, A_6, A_8, A_{10}, A_{12}, A_{14} \}$
8. bit-4-is-1 = $\{ A_1, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15} \}$

From each received symbol, R^k, the bit probabilities are computed as follows:

$$LLR(u_1^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}1\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_i \|\right)}{\sum_{A_j \in bit\text{-}1\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_j \|\right)}\right) \quad (28)$$

$$LLR(u_2^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}2\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_i \|\right)}{\sum_{A_j \in bit\text{-}2\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_j \|\right)}\right) \quad (29)$$

$$LLR(u_3^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}3\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_i \|\right)}{\sum_{A_j \in bit\text{-}3\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_j \|\right)}\right) \quad (30)$$

$$LLR(u_4^k) = \log\left(\frac{\sum_{A_i \in bit\text{-}4\text{-}is\text{-}1} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_i \|\right)}{\sum_{A_j \in bit\text{-}4\text{-}is\text{-}0} \exp\left(-\frac{1}{2\sigma_N^2}\| R^k - A_j \|\right)}\right) \quad (31)$$

9.4 Simulation Results

Figure 10 shows the simulation results for 5,120 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 11.8 dB.

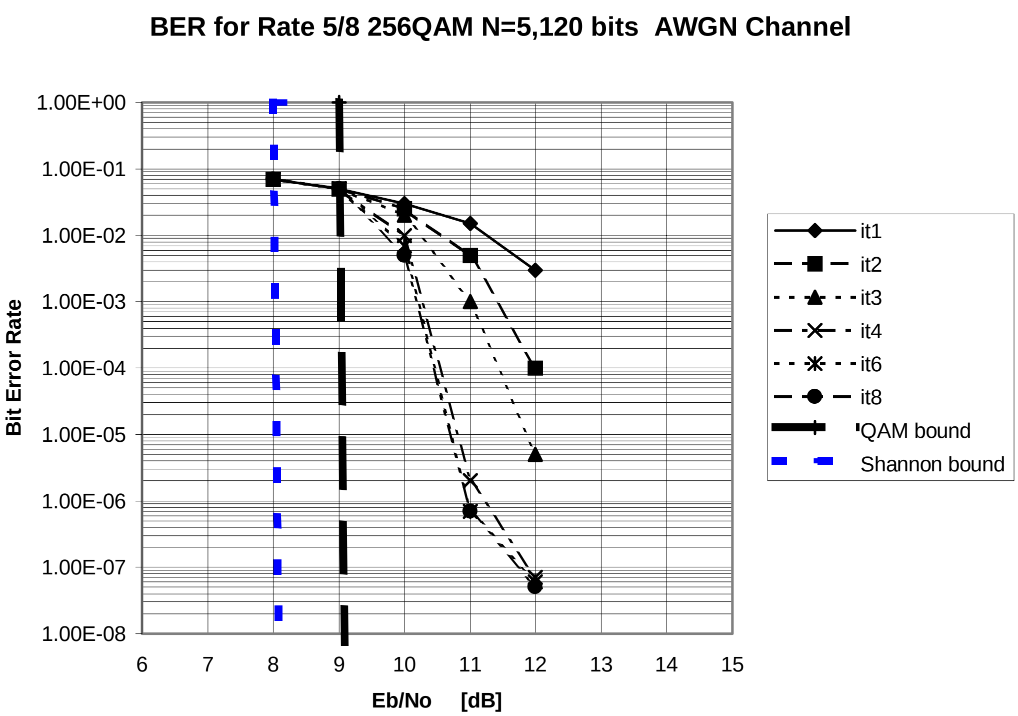

Figure 10

10. Coding And Modulation For 6 Bit/s/Hz Spectral Efficiency

This section investigates a rate 6/8 coding scheme with 256QAM.

10.1 Puncturing

In order to obtain a rate 6/8 code, the puncturing pattern used is shown in Table 9.

Table 9. Puncturing and Mapping for Rate 6/8 256QAM Option 1

Information bit (d)	d_1	d_2	d_3	d_4	d_5	d_6
parity bit (p)	p_1	-	-	-	-	-
parity bit (q)	-	-	-	q_4	-	-
16AM symbol (I)	(d_1, d_2, d_3, p_1)					
16AM symbol (Q)	(d_4, d_5, d_6, q_4)					
256QAM symbol (I, Q)	$(I, Q) = (d_1, d_2, d_3, p_1, d_4, d_5, d_6, q_4)$					

10.2 Modulation

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k)$ is sent though the channel. It is assumed that at time k the symbol u_1^k, u_2^k, u_3^k and u_4^k modulate the I component and u_5^k, u_6^k, u_7^k and u_8^k modulate the Q component of a 256QAM scheme.

For a rate 6/8 code and 256QAM, the noise variance is:

$$\sigma_N^2 = E_{av}\left(\frac{2\eta E_b}{N_0}\right)^{-1} = 170\,A^2\left(\frac{2\ x\ 6\ x\ E_b}{N_0}\right)^{-1} = 14.16\,A^2\left(\frac{E_b}{N_0}\right)^{-1} \tag{32}$$

The puncturing and mapping scheme shown in Table 9 is for 6 consecutive information bits that are coded into 8 coded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 6/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 6 bits/s/Hz.

10.3 Bit Probabilities

The 16AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k)$, where u_1^k is the most significant bit and u_4^k is the least significant bit. The following set can be defined.

1. bit-1-is-0 = $\{ A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7 \}$
2. bit-1-is-1 = $\{ A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15} \}$
3. bit-2-is-0 = $\{ A_0, A_1, A_2, A_3, A_8, A_9, A_{10}, A_{11} \}$
4. bit-2-is-1 = $\{ A_4, A_5, A_6, A_7, A_{12}, A_{13}, A_{14}, A_{15} \}$
5. bit-3-is-0 = $\{ A_0, A_1, A_4, A_5, A_8, A_9, A_{12}, A_{13} \}$
6. bit-3-is-1 = $\{ A_2, A_3, A_6, A_7, A_{10}, A_{11}, A_{14}, A_{15} \}$
7. bit-4-is-0 = $\{ A_0, A_2, A_4, A_6, A_8, A_{10}, A_{12}, A_{14} \}$
8. bit-4-is-1 = $\{ A_1, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15} \}$

From each received symbol, R^k, the bit probabilities are computed as equations (28) to (31).

10.4 Simulation Results

Figure 11 shows the simulation results for 6,144 information bits (1,204QAM symbols) with S-type interleaver. A BER of 10^{-7} can be achieved after 8 iterations at E_b/N_0 = 14.2 dB.

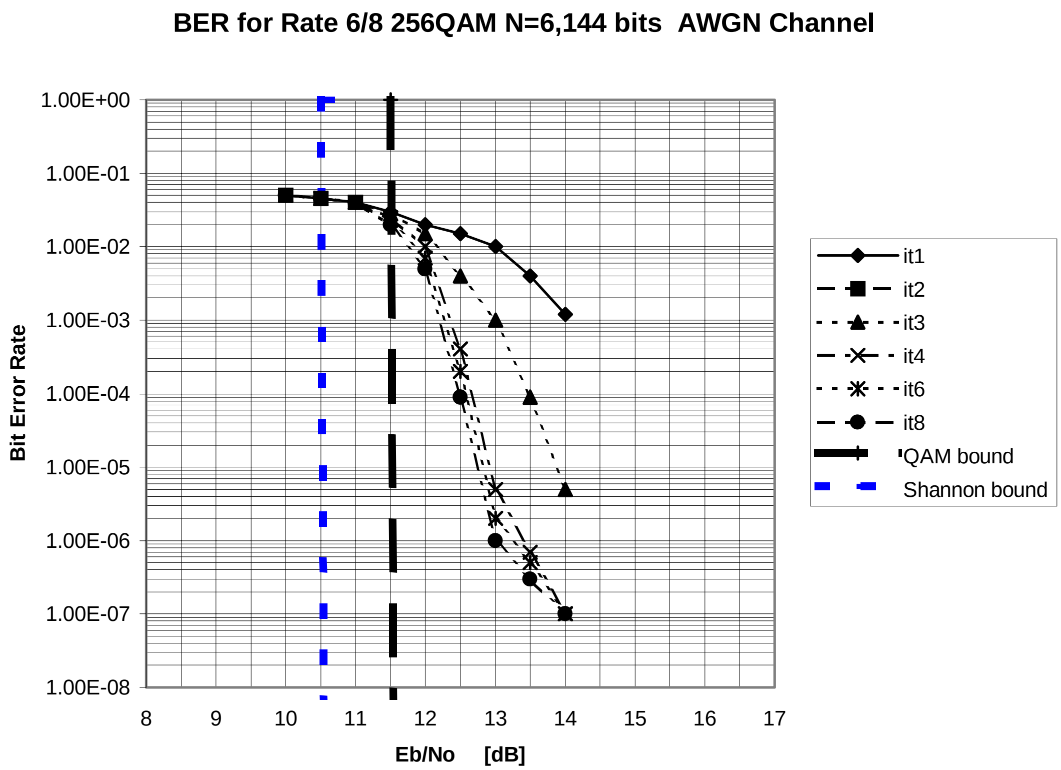

Figure 11

11. Power vs. Bandwidth In An AWGN Channel

This section gives an estimate of the trade off which can be achieved between minimum required E_b/N_0 and bandwidth efficiency. An information data rate of 2,048 Mbit/s and a maximum transmitter delay of 1 ms is considered. The corresponding interleaver size is 2,048 bits.

11.1 Channel model

All the simulations assumed the additive white Gaussian noise (AWGN) channel model, with independent I and Q signals.

11.2 Simulation Results

Simulations were run for bandwidth efficiencies from 1 to 7 bit/symbol using the recommended coding and modulation schemes. The results are shown in Figures 12 to 18.

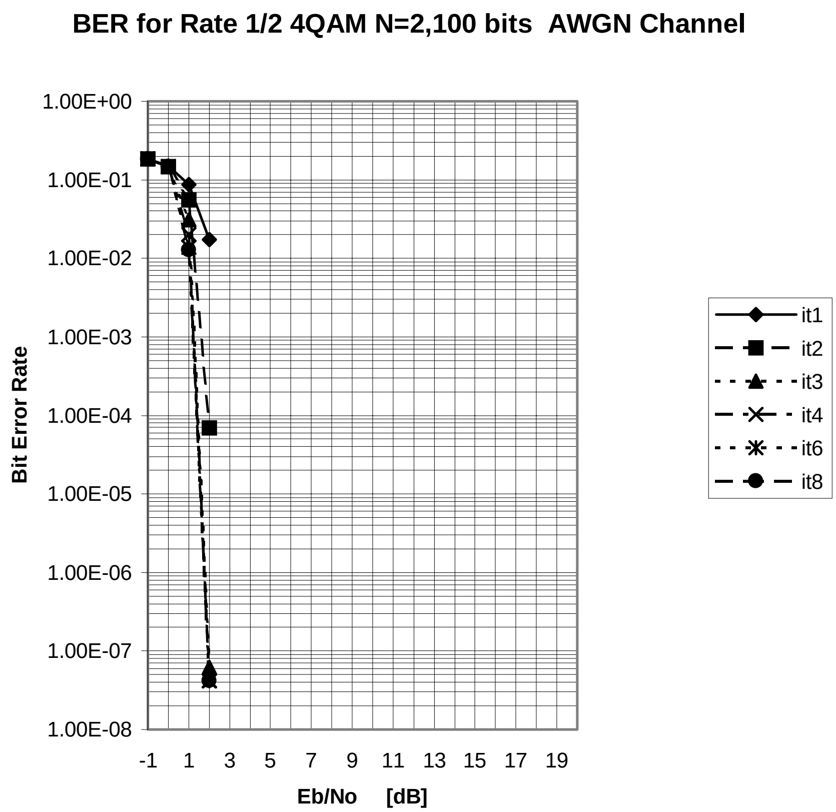

Figure 12

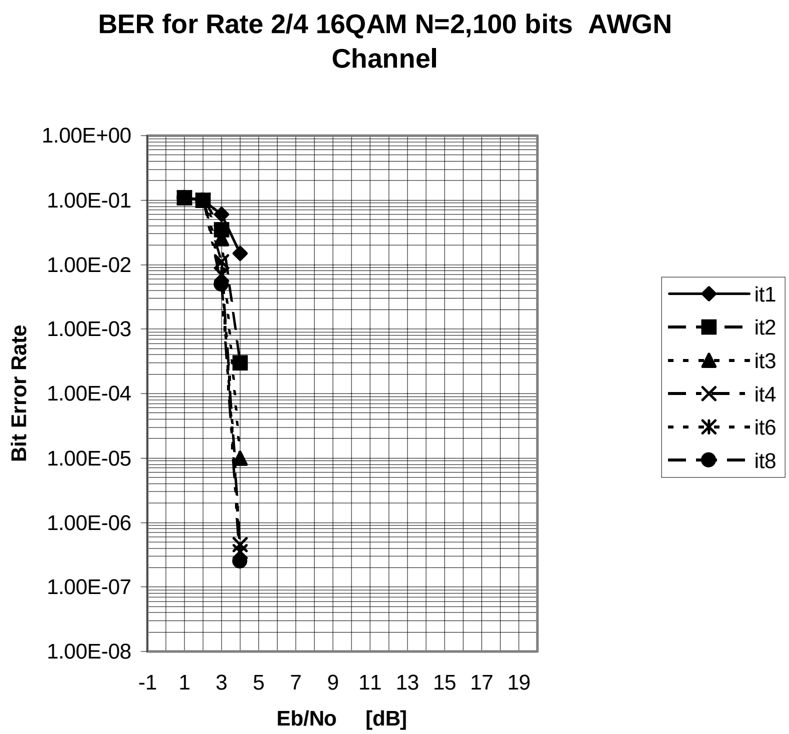

Figure 13

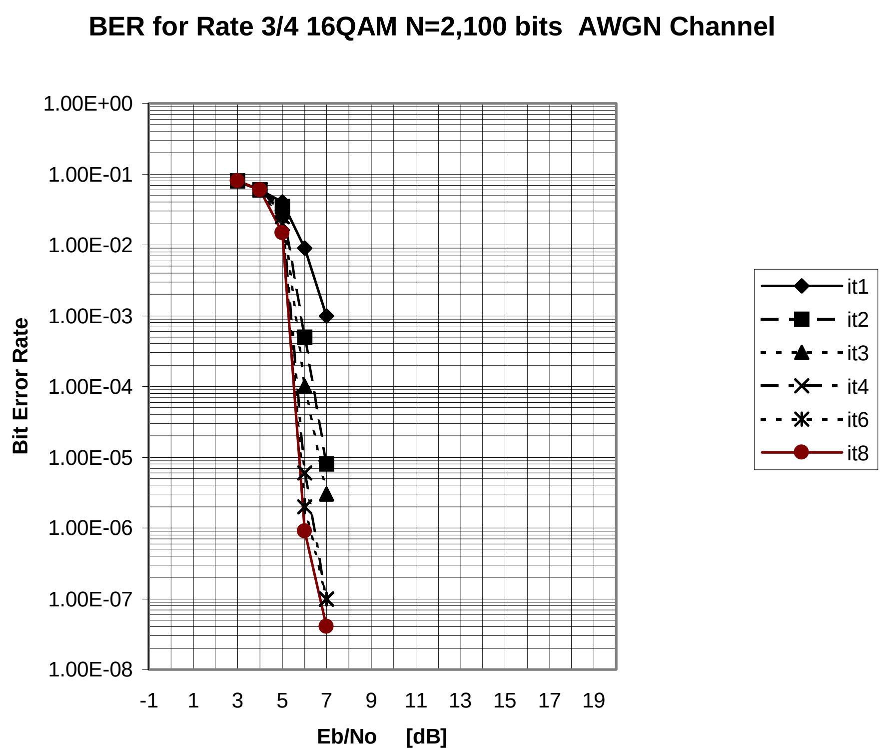

Figure 14

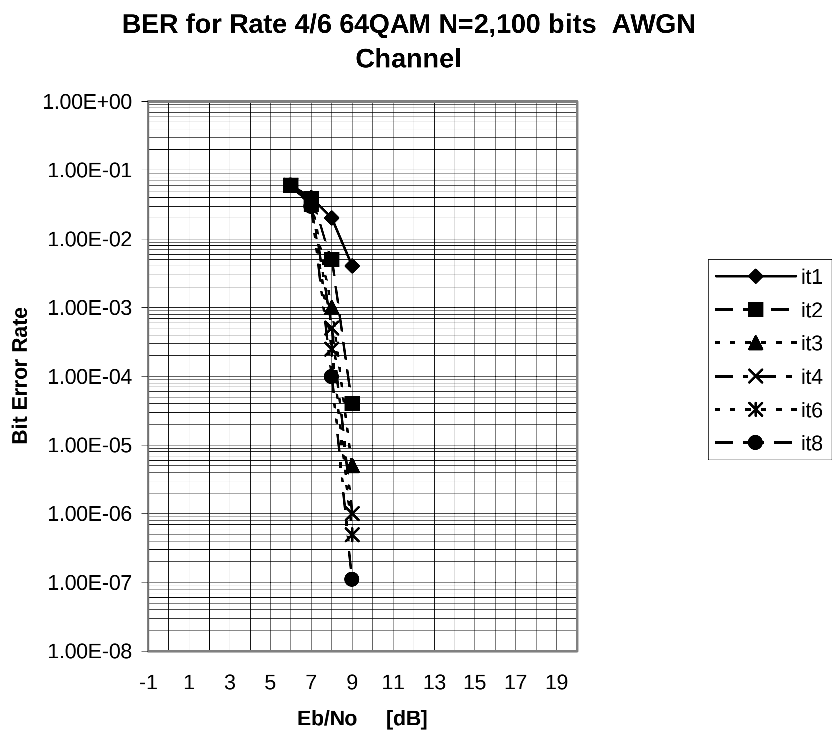

Figure 15

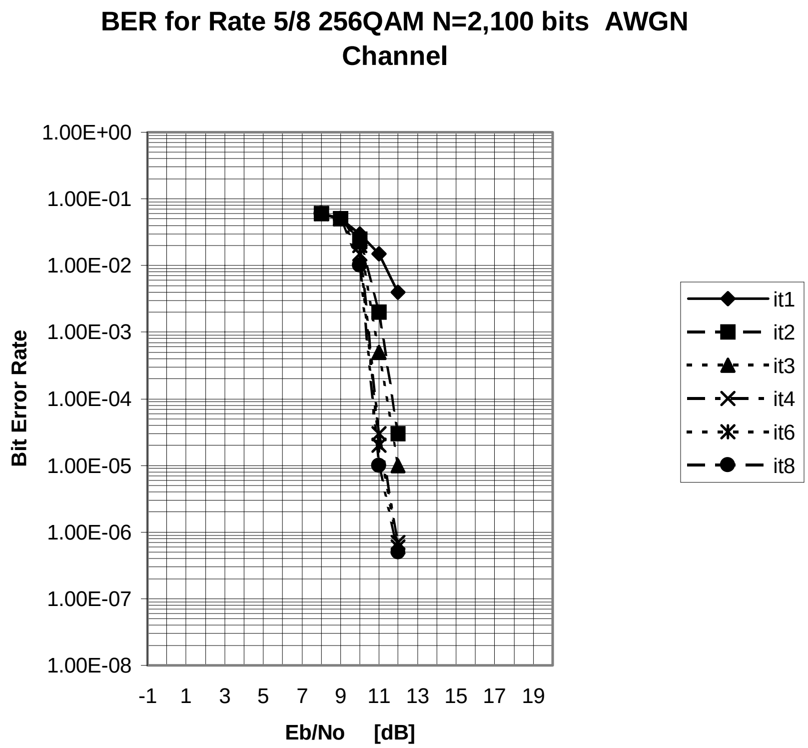

Figure 16

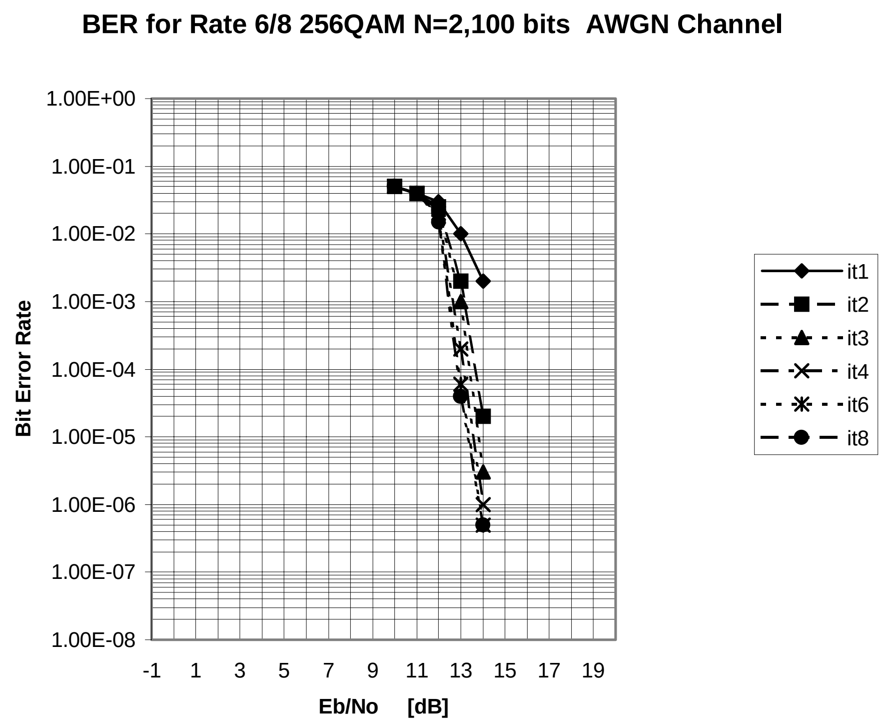

Figure 17

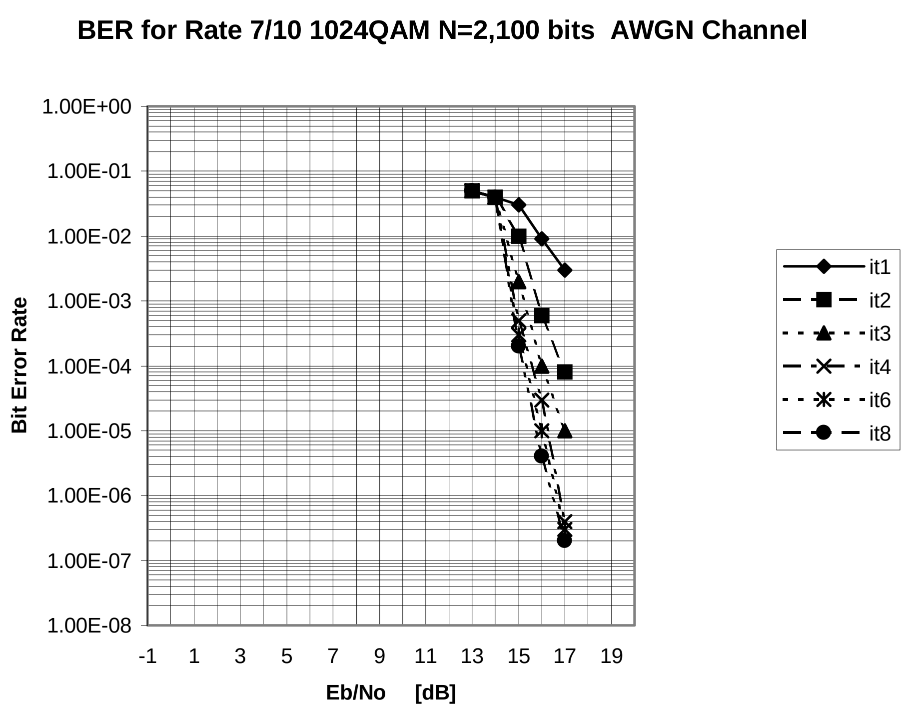

Figure 18

11.3 Conclusions

Table 10 summarizes the minimum E_b/N_0 required to achieve a BER of 10^{-7}.

Table 10 Minimum E_b/N_0 required to achieve a BER of 10^{-7}

Spectral efficiency η [bits/s/Hz]	Coding Rate And Modulation	E_b/N_0 For BER = 10^{-7} [dB]
1	1/2 and 4QAM	2.2
2	2/4 and 16QAM	4.2
3	3/4 and 16QAM	6.5
4	4/6 and 64QAM	9.1
5	5/8 and 256QAM	12.3
6	6/8 and 256QAM	14.5

The results show the potential reduction in bandwidth for a given signal-to-noise ration for a particular channel.

12. Conclusions

Table 11 summarizes all simulation results from this study.

Table 11. Summary of Some Simulation Results.

Spectral efficiency η[bits/s/Hz]	Coding Rate	Modulation	Interleaver size in Information bits	Required E_b/N_0[dB] BER= 10^{-7}	QAM bound [dB]
1	1/2	4QAM	1,024	2.1	1.0
2	2/4	16QAM	256	6.8	2.1
2	2/4	16QAM	512	5.3	2.1
2	2/4	16QAM	768	4.9	2.1
2	2/4	16QAM	1,024	4.5	2.1
2	2/4	16QAM	2,048	4.2	2.1
2	2/4	16QAM	32,728	2.9	2.1
3	3/4	16QAM	2,048	6.5	4.6
3	3/4	16QAM	4,096	5.8	4.6
3	3/6	64QAM	4,096	6.1	4.3
4	4/6	64QAM-1	4,096	8.3	6.6
5	5/6	64QAM	5,120	13.0	9.0
5	5/8	256QAM	2,048	12.3	9.0
5	5/8	256QAM	5,120	11.8	9.0
6	6/8	256QAM	2,048	14.5	11.7
6	6/8	256QAM	6,144	14.2	11.7