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<th><strong>Project</strong></th>
<th>IEEE 802.16 Broadband Wireless Access Working Group [<a href="http://ieee802.org/16">http://ieee802.org/16</a>]</th>
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<tr>
<td><strong>Title</strong></td>
<td>Method for using Concatenated Convolutional Turbo Codes in IEEE 802.16a</td>
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<tr>
<td><strong>Date Submitted</strong></td>
<td>2002-07-04</td>
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| **Source(s)** | Dr. Juan Alberto Torres  
VOCAL Technologies, Ltd.  
200 John James Audubon Parkway  
Buffalo, NY 14228 (http://www.vocal.com)  
Voice: 1 (716) 688-4675  
Fax: 1 (716) 639-0713  
mailto:jatorres@vocal.com |
| **Re:** | 802.16aD4 and 802.16cD1 |
| **Abstract** | This paper proposes a method for using concatenated convolutional Turbo codes for 802.16a. These concatenated convolutional turbo codes are optimized for each constellation size. |
| **Purpose** | To be used by TGa and TGc for discussion and to help preparing the draft document. |
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Juan Alberto Torres

Proposal

This paper proposes a method for using concatenated convolutional Turbo codes for 802.16a. These concatenated convolutional turbo codes are optimized for each constellation size.

Discussion

1. Introduction:

Turbo codes present a new and very powerful error control technique, which allows communication very close to the channel capacity. Since its discovery in 1993, a lot of research has been done in the application of turbo codes in deep space communications, mobile satellite/cellular communications, microwave links, paging, in OFDM and CDMA architectures. Turbo codes have outperformed all previously known coding schemes regardless of the targeted channel. The extra coding gain offered by turbo codes can be used either to minimize bandwidth or to reduce power requirements in the link budget. Standards based in turbo codes have already been defined or are currently under investigation. Here are some examples:

- Inmarsat’s new multimedia service is based on turbo codes and 16QAM that allows the user to communicate with existing Inmarsat 3 spot beam satellites from a notebook-sized terminal at 64 kbit/s.
- The Third Generation Partnership Project (3GPP) proposal for IMT-2000 includes turbo codes in the multiplexing and channel coding specification. The IMT-2000 represents the third generation mobile radio systems worldwide standard. The 3GPP objective is to harmonize similar standards proposals from Europe, Japan, Korea and the United States.
- NASA’s next-generation deep-space transponder will support turbo codes and implementation of turbo decoders in the Deep Space Network is planned by 2003.
- The new standard of the Consultative Committee for Space Data Systems (CCSDS) is based on turbo codes. The new standard outperforms by 1.5 to 2.8 dB the old CCSDS standard based on concatenated convolutional code and Reed-Solomon code.
- The new European Digital Video Broadcasting (DVB) standard has also adopted turbo codes for the return channel over satellite applications.

The way to avoid these computing requirements is to treat the QAM signal like two AM modulations (one in the I direction and one in the Q direction) and use the probabilities of the I and Q AM values as an input to the turbo code process.

Technique proposed here provides the most protected bits for the information bits and the least protected bits for the parity bits puncturing in applied to the parity bits.
2. Capacity Bounds

The minimum $E_b/N_0$ values to achieve the Shannon bound, 4QAM, 8QAM, 16QAM, 32QAM, 64QAM, 128QAM, 256QAM, 512QAM and 1024QAM bounds for spectral efficiencies from $2/3$ up to 7 bits/s/Hz respectively are as in Table 1 for a BER=$10^{-5}$.

Table 1. Shannon and QAM bounds.

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The conversion from $E_s/N_0$ to $E_b/N_0$ is performed using the following relation

$$E_b/N_0 [dB] = E_s/N_0 [dB] - 10 \log_{10} (\eta) [dB]$$

where $\eta$ is the number of information bits per symbol.

The required $C/N_0$ given a certain $E_b/N_0$ can be found using the following relation:

$$C/N_0 [dB-Hz] = E_b/N_0 [dB] + 10 \log_{10} (R_b) [dB-Hz]$$

where $R_b$ is the information bit rate.

For a D-dimension modulation the following formulae are used:

$$SNR = \frac{E[|a_i^1|^2]}{E[|w_k|^2]} = \frac{E[|a_i^1|^2]}{D\sigma_N^2} = \frac{E_o}{D\sigma_N^2}$$

$$SNR = \frac{E_o}{D\frac{N_0}{2}} = \eta \frac{E_o}{D\frac{N_0}{2}}$$

where $\sigma_N^2$ is the noise variance in each of the D dimension and $\eta$ is the number of information bits per symbol. From the above relations:

$$\sigma_N^2 = E_o \left( \frac{2\eta E_b}{N_0} \right)^{1/\eta}$$
3. Coding

The proposed coding scheme is shown in Figure 1. The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

4 Turbo code internal interleaver design (3GPP).

The Turbo code internal interleaver consists of bits-input to a rectangular matrix, intra-row and inter-row permutations of the rectangular matrix, and bits-output from the rectangular matrix with pruning. The bits input to the Turbo code internal interleaver are denoted by \( x_1, x_2, x_3, \ldots, x_K \), where \( K \) is the integer number of the bits and takes one value of \( 40 \leq K \leq 32000 \). The relation between the bits input to the Turbo code internal interleaver and the bits input to the channel coding is defined by \( x_k = o_{rk} \) and \( K = K_i \).

\[
\begin{align*}
K & \quad \text{Number of bits input to Turbo code internal interleaver} \\
R & \quad \text{Number of rows of rectangular matrix} \\
C & \quad \text{Number of columns of rectangular matrix} \\
p & \quad \text{Prime number} \\
v & \quad \text{Primitive root} \\
s(i) & \quad \text{Base sequence for intra-row permutation} \\
q_j & \quad \text{Minimum prime integers} \\
r_j & \quad \text{Permuted prime integers} \\
T(j) & \quad \text{Inter-row permutation pattern} \\
U_j(i) & \quad \text{Intra-row permutation pattern} \\
i & \quad \text{Index of matrix} \\
j & \quad \text{Index of matrix} \\
k & \quad \text{Index of bit sequence}
\end{align*}
\]

4.1 Bits-input to rectangular matrix

The bit sequence input to the Turbo code internal interleaver \( x_k \) is written into the rectangular matrix as follows.

(1) Determine the number of rows \( R \) of the rectangular matrix such that:
\[
R = \begin{cases} 
5, & \text{if } (40 \leq K \leq 159) \\
10, & \text{if } ((160 \leq K \leq 200) \text{ or } (481 \leq K \leq 530)) \\
20, & \text{if } (K = \text{any other value}) 
\end{cases}
\]

where the rows of rectangular matrix are numbered 0, 1, 2, ..., \( R - 1 \) from top to bottom.

(2) Determine the number of columns \( C \) of rectangular matrix such that:

if \((481 \leq K \leq 530)\) then 
\[
p = 53 \text{ and } C = p.
\]
else 
Find minimum prime \( p \) such that 
\[
(p + 1) - \frac{K}{R} \geq 0,
\]
and determine \( C \) such that 
if \((p - \frac{K}{R} \geq 0)\) then 
if \((p - 1 - \frac{K}{R} \geq 0)\) then 
\[
C = p - 1.
\]
else 
\[
C = p.
\]
end if 
else 
\[
C = p + 1
\]
end if 
where the columns of rectangular matrix are numbered 0, 1, 2, ..., \( C - 1 \) from left to right.

(3) Write the input bit sequence \( x_k \) into the \( R \times C \) rectangular matrix row by row starting with bit \( x_1 \) in column 0 of row 0:

\[
\begin{bmatrix}
x_1 & x_2 & x_3 & \cdots & x_C \\
x_{(C+1)} & x_{(C+2)} & x_{(C+3)} & \cdots & x_{2C} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{((R-1)C+1)} & x_{((R-1)C+2)} & x_{((R-1)C+3)} & \cdots & x_{RC}
\end{bmatrix}
\]
4.2 Intra-row and inter-row permutations

After the bits-input to the $R \times C$ rectangular matrix, the intra-row and inter-row permutations for the $R \times C$ rectangular matrix are performed by using the following algorithm.

1. Select a primitive root $v$ from table 2.

2. Construct the base sequence $s(i)$ for intra-row permutation as:

$$s(i) = [v \times s(i - 1)] \mod p, \quad i = 1, 2, ..., (p - 2), \text{ and } s(0) = 1.$$ 

3. Let $q_0 = 1$ be the first prime integer in $\{q_j\}$, and select the consecutive minimum prime integers $\{q_j\} (j = 1, 2, ..., R - 1)$ such that:

$$\text{g.c.d.}\{q_j, p - 1\} = 1, \quad q_j > 6, \text{ and } q_j > q_{j-1},$$

where $\text{g.c.d.}$ is greatest common divisor.

4. Permute $\{q_j\}$ to make $\{r_j\}$ such that

$$r_{T(j)} = q_j, \quad j = 0, 1, ..., R - 1,$$

where $T(j) (j = 0, 1, 2, ..., R - 1)$ is the inter-row permutation pattern defined as the one of the following four kind of patterns: $Pat_1, Pat_2, Pat_3$ and $Pat_4$ depending on the number of input bits $K$.

$$\{T(0), T(1), T(2), ..., T(R - 1)\} = \begin{cases} Pat_4 & \text{if } (40 \leq K \leq 159) \\ Pat_3 & \text{if } (160 \leq K \leq 200) \\ Pat_1 & \text{if } (201 \leq K \leq 480) \\ Pat_3 & \text{if } (481 \leq K \leq 530) \end{cases},$$

where $Pat_1, Pat_2, Pat_3$ and $Pat_4$ have the following patterns respectively.

$Pat_1$: $\{19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 10, 8, 13, 17, 3, 1, 16, 6, 15, 11\}$

$Pat_2$: $\{19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 16, 13, 17, 15, 3, 1, 6, 11, 8, 10\}$

$Pat_3$: $\{9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$

$Pat_4$: $\{4, 3, 2, 1, 0\}$

5. Perform the $j$-th ($j = 0, 1, 2, ..., R - 1$) intra-row permutation as:

if $(C = p)$ then
\[ U_j(i) = s([i \times r_j \mod (p - 1)], \quad i = 0, 1, 2, \ldots, (p - 2), \quad \text{and } U_j(p - 1) = 0, \]

where \( U_j(i) \) is the input bit position of \( i \)-th output after the permutation of \( j \)-th row.

end if

if \((C = p + 1)\) then

\[ U_j(i) = s([i \times r_j \mod (p - 1)], \quad i = 0, 1, 2, \ldots, (p - 2), \quad U_j(p - 1) = 0, \quad \text{and } U_j(p) = p, \]

where \( U_j(i) \) is the input bit position of \( i \)-th output after the permutation of \( j \)-th row, and

if \((K = C \times R)\) then

Exchange \( U_{R-1}(p) \) with \( U_{R-1}(0) \).

end

if

end if

end if

if \((C = p - 1)\) then

\[ U_j(i) = s([i \times r_j \mod (p - 1)]) - 1, \quad i = 0, 1, 2, \ldots, (p - 2), \]

where \( U_j(i) \) is the input bit position of \( i \)-th output after the permutation of \( j \)-th row.

end if

(6) Perform the inter-row permutation based on the pattern \( T(j) \) \((j = 0, 1, 2, \ldots, R - 1)\),
where \( T(j) \) is the original row position of the \( j \)-th permuted row.
Table 2: Table of prime $p$ and associated primitive root $v$

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<td>5</td>
<td>1439</td>
<td>7</td>
</tr>
<tr>
<td>193</td>
<td>5</td>
<td>571</td>
<td>3</td>
<td>991</td>
<td>6</td>
<td>1447</td>
<td>3</td>
</tr>
<tr>
<td>197</td>
<td>2</td>
<td>577</td>
<td>5</td>
<td>997</td>
<td>7</td>
<td>1451</td>
<td>2</td>
</tr>
<tr>
<td>199</td>
<td>3</td>
<td>587</td>
<td>2</td>
<td>1009</td>
<td>11</td>
<td>1453</td>
<td>2</td>
</tr>
<tr>
<td>211</td>
<td>2</td>
<td>593</td>
<td>3</td>
<td>1013</td>
<td>3</td>
<td>1459</td>
<td>3</td>
</tr>
<tr>
<td>223</td>
<td>3</td>
<td>599</td>
<td>7</td>
<td>1019</td>
<td>2</td>
<td>1471</td>
<td>6</td>
</tr>
<tr>
<td>227</td>
<td>2</td>
<td>601</td>
<td>7</td>
<td>1021</td>
<td>10</td>
<td>1481</td>
<td>3</td>
</tr>
<tr>
<td>229</td>
<td>6</td>
<td>607</td>
<td>3</td>
<td>1031</td>
<td>14</td>
<td>1483</td>
<td>2</td>
</tr>
<tr>
<td>233</td>
<td>3</td>
<td>613</td>
<td>2</td>
<td>1033</td>
<td>5</td>
<td>1487</td>
<td>5</td>
</tr>
<tr>
<td>239</td>
<td>7</td>
<td>617</td>
<td>3</td>
<td>1039</td>
<td>3</td>
<td>1489</td>
<td>14</td>
</tr>
<tr>
<td>241</td>
<td>7</td>
<td>619</td>
<td>2</td>
<td>1049</td>
<td>3</td>
<td>1493</td>
<td>2</td>
</tr>
<tr>
<td>251</td>
<td>6</td>
<td>631</td>
<td>3</td>
<td>1051</td>
<td>7</td>
<td>1499</td>
<td>2</td>
</tr>
<tr>
<td>257</td>
<td>3</td>
<td>641</td>
<td>3</td>
<td>1061</td>
<td>2</td>
<td>1511</td>
<td>11</td>
</tr>
<tr>
<td>263</td>
<td>5</td>
<td>643</td>
<td>11</td>
<td>1063</td>
<td>3</td>
<td>1523</td>
<td>2</td>
</tr>
<tr>
<td>269</td>
<td>2</td>
<td>647</td>
<td>5</td>
<td>1069</td>
<td>6</td>
<td>1531</td>
<td>2</td>
</tr>
</tbody>
</table>
4.3 Bits-output from rectangular matrix with pruning

After intra-row and inter-row permutations, the bits of the permuted rectangular matrix are denoted by $y'_k$:

$$
\begin{bmatrix}
  y'_1 & y'_{(R+1)} & y'_{(2R+1)} & \cdots & y'_{((C-1)R+1)} \\
  y'_2 & y'_{(R+2)} & y'_{(2R+2)} & \cdots & y'_{((C-1)R+2)} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  y'_{R} & y'_{2R} & y'_{3R} & \cdots & y'_{CR}
\end{bmatrix}
$$

The output of the Turbo code internal interleaver is the bit sequence read out column by column from the intra-row and inter-row permuted $R \times C$ matrix starting with bit $y'_1$ in row 0 of column 0 and ending with bit $y'_{CR}$ in row $R - 1$ of column $C - 1$. The output is pruned by deleting bits that were not present in the input bit sequence, i.e. bits $y'_k$ that correspond to bits $x_k$ with $k > K$ are removed from the output. The bits output from Turbo code internal interleaver are denoted by $x'_1, x'_2, \ldots, x'_K$, where $x'_1$ corresponds to the bit $y'_k$ with smallest index $k$ after pruning, $x'_2$ to the bit $y'_k$ with second smallest index $k$ after pruning, and so on. The number of bits output from Turbo code internal interleaver is $K$ and the total number of pruned bits is: $R \times C - K$.

5. Modulation For 1 Bit/s/Hz Spectral Efficiency.

The scheme proposed in this case combines a rate 1/2 coding scheme with 4QAM.

5.1 Puncturing

In order to obtain a code rate of 1/2, every other bit of the parity bits $p$ and $q$ from Figure 1 are punctured. The puncturing pattern is given in Table 3.

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>$d_1$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parity bit (p)</td>
<td>$p_1$</td>
<td>-</td>
</tr>
<tr>
<td>Parity bit (q)</td>
<td>-</td>
<td>$q_2$</td>
</tr>
<tr>
<td>2AM symbol (I)</td>
<td>$(u_1) = (d_1)$</td>
<td>$(u_1) = (d_2)$</td>
</tr>
<tr>
<td>2AM symbol (Q)</td>
<td>$(u_2) = (p_1)$</td>
<td>$(u_2) = (q_2)$</td>
</tr>
<tr>
<td>4QAM symbol (I, Q)</td>
<td>$(I, Q) = (u_1, u_2) = (d_1, p_1)$</td>
<td>$(I, Q) = (u_1, u_2) = (d_2, q_2)$</td>
</tr>
</tbody>
</table>
5.2. Modulation

A 4QAM scheme is shown in Figure 3. An equivalent 2AM modulation is shown in Figure 4.

At time $k$, the symbol $u^k = (u_1^k, u_2^k)$ is sent through the channel and the point $r^k$ in two dimensional space is received. For a 4QAM constellation with points at $-A$ and $A$, the $E_{av}$ is:

$$E_{av} = \frac{4(A^2 + A^2)}{4} = 2A^2$$  \hspace{1cm} (6)

For a rate $1/24$ code and 4QAM, the noise variance is:

$$\sigma_n^2 = E_{av} \left(\frac{2\eta E_b}{N_0}\right) = 2A^2 \left(\frac{2 \times 1 \times E_b}{N_0}\right) = A^2 \left(\frac{E_b}{N_0}\right)$$  \hspace{1cm} (7)

5.3. Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

$$LLR(u_1^i) = \log \left( \frac{\sum_{i=0}^{l} \exp \left( -\frac{1}{2\sigma_n^2} (t^k - a_{i1}^k)^2 \right) }{\sum_{i=0}^{l} \exp \left( -\frac{1}{2\sigma_n^2} (t^k - b_{i1}^k)^2 \right) } \right)$$

$$LLR(u_2^i) = \log \left( \frac{\sum_{i=0}^{l} \exp \left( -\frac{1}{2\sigma_n^2} (Q^k - a_{i1}^k)^2 \right) }{\sum_{i=0}^{l} \exp \left( -\frac{1}{2\sigma_n^2} (Q^k - b_{i1}^k)^2 \right) } \right)$$  \hspace{1cm} (8)

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 4 LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 2 bit-LLR values are reduced to one term each. Due to the puncturing, with one in two parity bits being transmitted, the expected performance will be lower when compared with the non-punctured scheme.

5.4. Simulations Results.

Figure 5 shows the performance of a turbo code using a 1024 bit S-type interleaver. The target BER of $10^{-7}$ for a 1,024 information bit interleaver can be achieve at $E_b/N_0 = 2.1$ dB.
6. Coding And Modulation For 2 Bit/s/Hz Spectral Efficiency.

The investigated combines 2/4 coding scheme with 16QAM.

6.1 Puncturing.

In order to obtain a rate 2/4 code, every other bit of the parity bits p and q from Figure 1 are punctured. The puncturing pattern is given in Table 4.

Table 4. Puncturing and Mapping for Rate 2/4 16QAM.

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>q₂</td>
</tr>
<tr>
<td>4AM symbol (I)</td>
<td>(u₁, u₂) = (d₁, p₁)</td>
<td></td>
</tr>
<tr>
<td>4AM symbol (Q)</td>
<td>(u₃, u₄) = (d₂, p₂)</td>
<td></td>
</tr>
<tr>
<td>16 QAM symbol (I, Q)</td>
<td>(I, Q) = (u₁, u₂, u₃, u₄) = (d₁, p₁, d₂, p₂)</td>
<td></td>
</tr>
</tbody>
</table>

6.2 Modulation

At time k, the symbol \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k) \) is sent through the channel and the point \( r^k \) in two dimensional space is received. For a 16QAM constellation with points at -3A, -A, A and 3A, The \( E_{av} \) is:
\[ E_{aw} = \frac{4(A^2 + A^2) + 8(A^2 + 9 A^2) + 4(9 A^2 + 9 A^2)}{16} = 10 A^2 \] (10)

For a rate 1/2 code and 16QAM, the noise variance in each dimension is:

\[ \sigma^2_k = E_{aw} \left( \frac{2\eta E_b}{N_0} \right) = 10 A^2 \left( \frac{2 \times 2 \times E_b}{N_0} \right)^i = 2.5 A^2 \left( \frac{E_b}{N_0} \right)^i \] (11)

It is assumed that the time \( k \), \( u_1^k \) and \( u_2^k \) modulate the I component and \( u_3^k \) and \( u_4^k \) modulate the Q component for a 16QAM scheme. The symbol \( u^k \) symbol has the following mapping: \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k) = (d_1^k, p_1^k, d_3^k, q_3^k) \). The parity bits are mapped to the least protected bits of the QAM symbol. Note that \( k \) denotes the symbol time index and \( i \) the information bit time index. This means a puncturing of one in two parity bits. Considering two independent Gaussian noise sources with identical variance \( \sigma^2_N \), the LLR can be determined independently for each I and Q.

At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the 4 bit-LLR values. The mapping of the information bit is made to the most protected bit in each dimension (\( u_1^k \) for the I signal and \( u_3^k \) for the Q signal). In order to estimate the performance of this scheme, rate 1/2 turbo code and 4AM modulation is used, instead of 16QAM modulation.

6.3 Bit probabilities.

For an AWGN channel the following expressions need to be evaluated:

\[
\text{LLR}(u_1^k) = \log \frac{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - a_i^k)^2\right) I}{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - a_i^k)^2\right) R} = \log \frac{\exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right) + \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right)}{\exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right)}
\] (12)

\[
\text{LLR}(u_2^k) = \log \frac{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - a_i^k)^2\right) I}{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - a_i^k)^2\right) R} = \log \frac{\exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right) + \exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right)}{\exp\left(-\frac{1}{2 \sigma_N^2} (t_i^k - A_i^k)^2\right)}
\] (13)

\[
\text{LLR}(u_3^k) = \log \frac{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - a_i^k)^2\right) I}{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - a_i^k)^2\right) R} = \log \frac{\exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right) + \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right)}{\exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right)}
\] (14)

\[
\text{LLR}(u_4^k) = \log \frac{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - a_i^k)^2\right) I}{\sum_{i=1}^{2} \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - a_i^k)^2\right) R} = \log \frac{\exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right) + \exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right)}{\exp\left(-\frac{1}{2 \sigma_N^2} (q_i^k - B_i^k)^2\right)}
\] (15)

The above LLRs are used as inputs to the turbo decoder. There is no need to compute the 16LLRs for all symbols because I and Q signals are treated independently. Also the simulations for the 4 bit-LLR values are reduced to 2 terms each. Due to the puncturing, with one in two parity bits being
transmitted, the expected performance will be lower when compared with the non punctured scheme.

6.4 Simulations Results.

The rate 2/4 16QAM scheme described in this chapter achieves the target BER at less than 1.5 dB from capacity. The implementation in hardware is feasible and it can be used at very high data rates. The target BER of $10^{-7}$ can be achieved at $E_b/N_0 = 4.5$ dB for $N=1024$ information bits, as shown in Figure 6.

![BER for Rate 2/4 16QAM, N=1024 bits S-type, AWGN Channel](image)

Figure 6

7. Coding And Modulation For 3 Bit/s/Hz Spectral Efficiency.

Two options are investigated in this section. The first scheme combines a rate 3/4 coding scheme with 16QAM. The second scheme combines a rate 3/4 coding scheme with 64QAM.

7.1 Option 1 Rate 3/4 Turbo code and 16QAM

7.1.1 Puncturing.

In order to obtain a rate 3/4 code, the puncturing pattern given in Table 5 is used.
Table 5. Puncturing and Mapping for Rate 3/4 16QAM

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
<th>d₅</th>
<th>d₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>-</td>
<td>p₂</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>q₅</td>
<td>-</td>
</tr>
<tr>
<td>4AM symbol (I)</td>
<td>(d₁, d₂)</td>
<td>(d₄, d₅)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4AM symbol (Q)</td>
<td>(d₃, p₂)</td>
<td>(d₆, p₅)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 QAM symbol (I, Q)</td>
<td>(I, Q) = (d₁, d₂, d₃, p₂)</td>
<td>(I, Q) = (d₄, d₅, d₆, q₅)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.1.2 Modulation

It is assumed that at time k, u₁ₖ and u₂ₖ modulate the I component and u₃ₖ and u₄ₖ modulate the Q component of a 16QAM scheme. In order to estimate the performance of this scheme, rate 3/4 turbo codes and 4 AM modulation are used. For a rate 3/4 code and 16QAM, noise variance is:

\[
\sigma^2 = E_{av} \left( \frac{2 \eta E_b}{N_o} \right)^{1/2} = 10 A^2 \left( \frac{2 x 3 x E_b}{N_o} \right)^{1/2} = \frac{10}{6} A^2 \left( \frac{E_b}{N_o} \right)^{1/2}
\] (16)

The puncturing and mapping scheme is shown in Table 5 for 6 consecutive information bits that are encoded into 8 coded bits, therefore two 16QAM symbols.

7.1.3 Bit Probabilities

For each received symbol, the bit probabilities are computed as described in equations (12)-(15).

7.1.4 Simulation Results

Figure 7 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10⁻⁷ can be achieved after 8 iterations at \( E_b/N_o = 5.75 \) dB for \( N = 6144 \) information bits.

7.2 Option 2 Rate 3/6 Turbo code and 64QAM

7.2.1 Puncturing.

In order to obtain a rate 3/6 code, the puncturing pattern used is given in Table 6.

Table 6. Puncturing and Mapping for Rate 3/6 64QAM

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
<th>d₅</th>
<th>d₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td>-</td>
<td>p₃</td>
<td>-</td>
<td>p₅</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>q₂</td>
<td>-</td>
<td>q₄</td>
<td>-</td>
<td>q₆</td>
</tr>
<tr>
<td>8AM symbol (I)</td>
<td>(d₁, d₂, p₁)</td>
<td>(d₄, d₅, q₄)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8AM symbol (Q)</td>
<td>(d₃, p₃, q₂)</td>
<td>(d₆, p₅, q₆)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 QAM symbol (I, Q)</td>
<td>(I, Q) = (d₁, d₂, p₁, d₃, p₃, q₂)</td>
<td>(I, Q) = (d₄, d₅, q₄, d₆, p₅, q₆)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
BER for Rate 3/4 16QAM, N=6,144 bits S-type, AWGN Channel

Figure 7

7.2.2 Modulation

At time k, the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k)$, is sent through the channel and the point $r^k$ in two dimensional space is received. It is assumed that at time k $u_1^k$, $u_2^k$ and $u_3^k$ modulate the I component and $u_4^k$, $u_5^k$ and $u_6^k$ modulate the Q component of a 64QAM scheme. For 64QAM constellations with points at -7A, -5A, -3A, -A, A, 3A, 5A, 7A The $E_{av}$ is:

$$E_{av} = (8(49+25+9+1)+8(25+49+49+49+1)+8(25+9+25+1)+8(9+1)) A^2 / 64 = 42 A^2$$  \hspace{1cm} (17)

For a rate 3/6 code and 64QAM, the noise variance in each dimension is:

$$\sigma^2_n = E_{av} \left( \frac{2\eta E_b}{N_0} \right)^{ci} = 42 A^{i} \left( \frac{2x3x E_b}{N_0} \right)^{ci} = 7 A^{i} \left( \frac{E_b}{N_0} \right)^{ci}$$  \hspace{1cm} (18)

In order to estimate the performance of this scheme, when rate 3/6 turbo codes and 8AM modulation is used. The puncturing and mapping scheme is shown in Table 6 for 6 consecutive information bits that are encoded into 12 coded bits, therefore two 64QAM symbols.
7.2.3 Bit Probabilities

For an AWGN channel, the following expressions need to be evaluated for the I dimension:

\[
LLR(u_i) = \log \frac{\sum_{i=1}^{4} \exp[-\frac{1}{2\sigma_N^2} (I^i - a_{i0}^k)^2]}{\sum_{i=1}^{4} \exp[-\frac{1}{2\sigma_N^2} (I^i - a_{i0}^k)^2]}
\]

(19)

\[
= \log \left( \frac{\exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2]}{\exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2]} \right)
\]

\[
LLR(u_i^2) = \log \frac{\sum_{i=1}^{4} \exp[-\frac{1}{2\sigma_N^2} (I^i - a_{i0}^k)^2]}{\sum_{i=1}^{4} \exp[-\frac{1}{2\sigma_N^2} (I^i - a_{i0}^k)^2]}
\]

(20)

\[
= \log \left( \frac{\exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2]}{\exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2] + \exp[-\frac{1}{2\sigma_N^2} (I^i - A_i)^2]} \right)
\]

(21)

An identical computation effort is required for the Q dimension, the I\textsuperscript{k} being replaced with the Q\textsuperscript{k} demodulated value in order to evaluate LLR(u\textsubscript{q}\textsuperscript{k}), LLR(u\textsubscript{q}\textsuperscript{5}) and LLR(u\textsubscript{q}\textsuperscript{6}).

7.2.4 Simulation Results

Figure 8 shows the simulation results for 6,144 information bits with S-type interleaver. A BER of 10\textsuperscript{-7} can be achieved after 8 iterations at \(E_b/N_0 = 6.1\) dB for \(N = 6,144\) information bits. This result is 0.5 dB worse than the performance of the rate 3/4 16QAM scheme.
8. Coding And Modulation For 4 Bit/s/Hz Spectral Efficiency.

This scheme uses independent I and Q mapping and also uses Gray mapping in each dimension.

8.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing pattern used is shown in Table 7.

Table 7. Puncturing and Mapping for Rate 4/6 64QAM Option 1

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d_1</th>
<th>d_2</th>
<th>d_3</th>
<th>d_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p_1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>-</td>
<td>q_3</td>
<td>-</td>
</tr>
<tr>
<td>8AM symbol (I)</td>
<td>(d_1, d_2, p_1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8AM symbol (Q)</td>
<td>(d_3, d_4, q_3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 QAM symbol (I, Q)</td>
<td>(I, Q)=(d_1, d_2, p_1, d_3, d_4, q_3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8.2 Modulation

Gray mapping was used in each dimension. Four information bits are required to be sent using a 64QAM constellation. For a rate 4/6 code and 64QAM, the noise variance in each dimension is

\[
\sigma^2_N = E_{aw} \left( \frac{2\eta E_b}{N_0} \right)^2 = 42 A^2 \left( \frac{2 \times 4 \times E_b}{N_0} \right)^2 = 5.25 A^2 \left( \frac{E_b}{N_0} \right)^2
\]  

(22)

The puncturing and mapping scheme is shown in Table 7 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64QAM symbol. The turbo encoder with the puncturing presented in Table 7 is a rate 4/6 turbo code which in conjunction with 64QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance \( \sigma^2_N \), the LLR can be determined independently for each I and Q. It is assumed that at time \( k \) \( u_1^k, u_2^k \) and \( u_3^k \) modulate the I component and \( u_4^k, u_5^k \) and \( u_6^k \) modulate the Q component of the 64QAM scheme. At the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

8.3 Bit Probabilities

From each received symbol, the bit probabilities are computed as follows:

\[
LLR(u_i^k) = \log \left( \frac{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]}{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]} \right)
\]

(23)

\[
LLR(u_i^k) = \log \left( \frac{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]}{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]} \right)
\]

(24)

\[
LLR(u_i^k) = \log \left( \frac{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]}{\sum_{i=1}^4 \exp \left[ -\frac{1}{2\sigma^2_N} (j^i - a_i^k)^2 \right]} \right)
\]

(25)

For I dimension. An identical computation effort is required for the Q dimension, the \( I^k \) being replaced with the \( Q^k \) demodulated value in order to evaluate \( LLR(u_4^k), LLR(u_5^k) \) and \( LLR(u_6^k) \).
8.4 Simulation Results

Figure 9 shows the simulation results for 4,096 information bits with S-type interleaver. A BER of $10^{-7}$ can be achieved after 8 iterations at $E_b/N_0 = 8.3$ dB.

![BER for Rate 4/6 64QAM N=4,096 bits AWGN Channel, Gray Mapping](image)

Figure 9.

9. Coding And Modulation For 5 Bit/s/Hz Spectral Efficiency.

This section investigated a rate 5/8 coding scheme with 256QAM.

9.1 Puncturing

In order to obtain a rate 5/8 code, the puncturing pattern used is shown in Table 8.
Table 8. Puncturing and Mapping for Rate 5/8 256QAM

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
<th>d₅</th>
<th>d₆</th>
<th>d₇</th>
<th>d₈</th>
<th>d₉</th>
<th>d₁₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>p₅</td>
<td>-</td>
<td>-</td>
<td>p₈</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>parity bit (q)</td>
<td>-</td>
<td>-</td>
<td>q₃</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>q₆</td>
<td>-</td>
<td>-</td>
<td>q₁₀</td>
</tr>
<tr>
<td>16AM symbol (I)</td>
<td>(d₁, d₂, d₃, p₁)</td>
<td>(d₆, d₇, d₈, q₆)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16AM symbol (Q)</td>
<td>(d₄, d₅, q₃, p₅)</td>
<td>(d₉, d₁₀, p₈, q₁₀)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>256 QAM symbol (I, Q)</td>
<td>(d₁, d₂, d₃, p₁, d₄, d₅, q₃, p₅)</td>
<td>(d₆, d₇, d₈, q₆, d₉, d₁₀, p₈, q₁₀)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9.2 Modulation


\[
E_{av} = 170 A^2
\]  \hspace{1cm} (26)

It is assumed that at time \( k \) the symbol \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k) \) is sent through the channel. It is assumed that at time \( k \) the symbol \( u_1^k, u_2^k, u_3^k \) and \( u_4^k \) modulate the I component and \( u_5^k, u_6^k, u_7^k \) and \( u_8^k \) modulate the Q component of a 256QAM scheme.

For a rate 5/8 code and 256QAM, the noise variance is:

\[
\sigma^2 = E_{av} \left( \frac{2\eta E_b}{N_0} \right) = 170 A^2 \left( \frac{2 \times 5 \times E_b}{N_0} \right) = 17 A^2 \left( \frac{E_b}{N_0} \right)
\]  \hspace{1cm} (27)

In order to study the performance of this scheme, a rate 5/8 turbo code and a 16AM is used. The 256QAM scheme will achieve a similar performance in terms of bit error rate (BER) at twice the spectral efficiency, assuming an ideal demodulator. The puncturing and mapping scheme shown in Table 8 is for 10 consecutive information bits that are coded into 16 encoded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 5/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 5 bits/s/Hz.

9.3 Bit Probabilities

The 16AM symbol is defined as \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k) \), where \( u_1^k \) is the most significant bit and \( u_4^k \) is the least significant bit. The following set can be defined.

1. bit-1-is-0 = \{ A₀, A₁, A₂, A₃, A₄, A₅, A₆, A₇ \}
2. bit-1-is-1 = \{ A₈, A₉, A₁₀, A₁₁, A₁₂, A₁₃, A₁₄, A₁₅ \}
3. bit-2-is-0 = \{ A₀, A₁, A₂, A₃, A₈, A₉, A₁₀, A₁₁ \}
4. bit-2-is-1 = \{ A₄, A₅, A₆, A₇, A₁₂, A₁₃, A₁₄, A₁₅ \}
5. bit-3-is-0 = \{ A₀, A₁, A₄, A₅, A₈, A₉, A₁₂, A₁₃ \}
6. bit-3-is-1 = \{ A₂, A₃, A₆, A₇, A₁₀, A₁₁, A₁₄, A₁₅ \}
7. bit-4-is-0 = \{ A₀, A₂, A₄, A₆, A₈, A₁₀, A₁₂, A₁₄ \}
8. bit-4-is-1 = \{ A₁, A₃, A₅, A₇, A₉, A₁₁, A₁₃, A₁₅ \}

From each received symbol, \( R^k \), the bit probabilities are computed as follows:
\[ LLR(u_i^k) = \log \left( \frac{\sum_{A_i \in b_{i-1}+i} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)}{\sum_{A_i \in b_{i-1}+i+1} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)} \right) \] (28)

\[ LLR(u_i^k) = \log \left( \frac{\sum_{A_i \in b_{i-2}+i} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)}{\sum_{A_i \in b_{i-2}+i+1} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)} \right) \] (29)

\[ LLR(u_i^k) = \log \left( \frac{\sum_{A_i \in b_{i-3}+i} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)}{\sum_{A_i \in b_{i-3}+i+1} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)} \right) \] (30)

\[ LLR(u_i^k) = \log \left( \frac{\sum_{A_i \in b_{i-4}+i} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)}{\sum_{A_i \in b_{i-4}+i+1} \exp \left( -\frac{1}{2\sigma_N^2} \| R^k - A_i \| \right)} \right) \] (31)

9.4 Simulation Results

Figure 10 shows the simulation results for 5,120 information bits (1,204QAM symbols) with S-type interleaver. A BER of $10^{-7}$ can be achieved after 8 iterations at $E_b/N_0 = 11.8$ dB.
10. Coding And Modulation For 6 Bit/s/Hz Spectral Efficiency

This section investigates a rate 6/8 coding scheme with 256QAM.

10.1 Puncturing

In order to obtain a rate 6/8 code, the puncturing pattern used is shown in Table 9.

Table 9. Puncturing and Mapping for Rate 6/8 256QAM Option 1

<table>
<thead>
<tr>
<th>Information bit (d)</th>
<th>d₁</th>
<th>d₂</th>
<th>d₃</th>
<th>d₄</th>
<th>d₅</th>
<th>d₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity bit (p)</td>
<td>p₁</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parity bit (q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>q₄</td>
<td></td>
</tr>
<tr>
<td>16AM symbol (I)</td>
<td></td>
<td>(d₁, d₂, d₃, p₁)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16AM symbol (Q)</td>
<td></td>
<td>(d₄, d₅, d₆, q₄)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>256QAM symbol (I, Q)</td>
<td>(I, Q) = (d₁,d₂, d₃, p₁, d₄,d₅, d₆, q₄)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10.2 Modulation

It is assumed that at time k the symbol \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k) \) is sent through the channel. It is assumed that at time k the symbol \( u_1^k, u_2^k, u_3^k \) and \( u_4^k \) modulate the I component and \( u_5^k, u_6^k, u_7^k \) and \( u_8^k \) modulate the Q component of a 256QAM scheme.

For a rate 6/8 code and 256QAM, the noise variance is:

\[
\sigma^2_N = E_{\text{x}} \left( \frac{2\eta E_b}{N_0} \right) ^2 = 170 A^2 \left( \frac{2 \times 6 \times E_b}{N_0} \right) ^2 = 14.16 A^2 \left( \frac{E_b}{N_0} \right) ^2 \quad (32)
\]

The puncturing and mapping scheme shown in Table 9 is for 6 consecutive information bits that are coded into 8 coded bits, therefore, one 256QAM symbol. The turbo encoder is a rate 6/8 turbo code, which in conjunction with 256QAM, gives a spectral efficiency of 6 bits/s/Hz.

10.3 Bit Probabilities

The 16AM symbol is defined as \( u^k = (u_1^k, u_2^k, u_3^k, u_4^k) \), where \( u_1^k \) is the most significant bit and \( u_4^k \) is the least significant bit. The following set can be defined.

1. bit-1-is-0 = \{ A₀, A₁, A₂, A₃, A₄, A₅, A₆, A₇ \}
2. bit-1-is-1 = \{ A₈, A₉, A₁₀, A₁₁, A₁₂, A₁₃, A₁₄, A₁₅ \}
3. bit-2-is-0 = \{ A₀, A₁, A₂, A₃, A₄, A₅, A₆, A₉, A₁₀, A₁₁ \}
4. bit-2-is-1 = \{ A₄, A₅, A₆, A₇, A₁₂, A₁₃, A₁₄, A₁₅ \}
5. bit-3-is-0 = \{ A₀, A₁, A₄, A₅, A₆, A₉, A₁₂, A₁₃ \}
6. bit-3-is-1 = \{ A₂, A₃, A₆, A₇, A₁₀, A₁₁, A₁₄, A₁₅ \}
7. bit-4-is-0 = \{ A₀, A₂, A₄, A₆, A₈, A₁₀, A₁₂, A₁₄ \}
8. bit-4-is-1 = \{ A₁, A₃, A₅, A₇, A₉, A₁₁, A₁₃, A₁₅ \}

From each received symbol, \( R^k \), the bit probabilities are computed as equations (28) to (31).
10.4 Simulation Results

Figure 11 shows the simulation results for 6,144 information bits (1,204QAM symbols) with S-type interleaver. A BER of $10^{-7}$ can be achieved after 8 iterations at $E_b/N_0 = 14.2$ dB.
11. Power vs. Bandwidth In An AWGN Channel

This section gives an estimate of the trade off which can be achieved between minimum required $E_b/N_0$ and bandwidth efficiency. An information data rate of 2,048 Mbit/s and a maximum transmitter delay of 1 ms is considered. The corresponding interleaver size is 2,048 bits.

11.1 Channel model

All the simulations assumed the additive white Gaussian noise (AWGN) channel model, with independent I and Q signals.

11.2 Simulation Results

Simulations were run for bandwidth efficiencies from 1 to 7 bit/symbol using the recommended coding and modulation schemes. The results are shown in Figures 12 to 18.

Figure 12
Figure 13

BER for Rate 3/4 16QAM N=2,100 bits  AWGN Channel

Figure 14
BER for Rate 4/6 64QAM N=2,100 bits  AWGN

Channel

Bit Error Rate

Figure 15

BER for Rate 5/8 256QAM N=2,100 bits  AWGN

Channel

Bit Error Rate

Figure 16
BER for Rate 6/8 256QAM N=2,100 bits  AWGN Channel

Figure 17

BER for Rate 7/10 1024QAM N=2,100 bits  AWGN Channel

Figure 18
11.3 Conclusions

Table 10 summarizes the minimum $E_b/N_0$ required to achieve a BER of $10^{-7}$.

Table 10 Minimum $E_b/N_0$ required to achieve a BER of $10^{-7}$

<table>
<thead>
<tr>
<th>Spectral efficiency $\eta$ [bits/s/Hz]</th>
<th>Coding Rate And Modulation</th>
<th>$E_b/N_0$ For BER $= 10^{-7}$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2 and 4QAM</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>2/4 and 16QAM</td>
<td>4.2</td>
</tr>
<tr>
<td>3</td>
<td>3/4 and 16QAM</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>4/6 and 64QAM</td>
<td>9.1</td>
</tr>
<tr>
<td>5</td>
<td>5/8 and 256QAM</td>
<td>12.3</td>
</tr>
<tr>
<td>6</td>
<td>6/8 and 256QAM</td>
<td>14.5</td>
</tr>
</tbody>
</table>

The results show the potential reduction in bandwidth for a given signal-to-noise ratio for a particular channel.

12. Conclusions

Table 11 summarizes all simulation results from this study.

Table 11. Summary of Some Simulation Results.

<table>
<thead>
<tr>
<th>Spectral efficiency $\eta$[bits/s/Hz]</th>
<th>Coding Rate</th>
<th>Modulation</th>
<th>Interleaver size in Information bits</th>
<th>Required $E_b/N_0$[dB] BER$= 10^{-7}$</th>
<th>QAM bound [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>4QAM</td>
<td>1,024</td>
<td>2.1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>256</td>
<td>6.8</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>512</td>
<td>5.3</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>768</td>
<td>4.9</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>1,024</td>
<td>4.5</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>2,048</td>
<td>4.2</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>2/4</td>
<td>16QAM</td>
<td>32,728</td>
<td>2.9</td>
<td>2.1</td>
</tr>
<tr>
<td>3</td>
<td>3/4</td>
<td>16QAM</td>
<td>2,048</td>
<td>6.5</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>3/4</td>
<td>16QAM</td>
<td>4,096</td>
<td>5.8</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>3/6</td>
<td>64QAM</td>
<td>4,096</td>
<td>6.1</td>
<td>4.3</td>
</tr>
<tr>
<td>4</td>
<td>4/6</td>
<td>64QAM-1</td>
<td>4,096</td>
<td>8.3</td>
<td>6.6</td>
</tr>
<tr>
<td>5</td>
<td>5/6</td>
<td>64QAM</td>
<td>5,120</td>
<td>13.0</td>
<td>9.0</td>
</tr>
<tr>
<td>5</td>
<td>5/8</td>
<td>256QAM</td>
<td>2,048</td>
<td>12.3</td>
<td>9.0</td>
</tr>
<tr>
<td>5</td>
<td>5/8</td>
<td>256QAM</td>
<td>5,120</td>
<td>11.8</td>
<td>9.0</td>
</tr>
<tr>
<td>6</td>
<td>6/8</td>
<td>256QAM</td>
<td>2,048</td>
<td>14.5</td>
<td>11.7</td>
</tr>
<tr>
<td>6</td>
<td>6/8</td>
<td>256QAM</td>
<td>6,144</td>
<td>14.2</td>
<td>11.7</td>
</tr>
</tbody>
</table>