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Title	**Reasons-to-use-non-squared-qam-constellations-with-independent-i&q-in-pan-systems** (http://www.vocal.com)
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Re:	[Response to call for contribution made on 9/03 at 3:33pm]
Abstract	[This paper provides a rational for the use of non-squared constellations in PAN systems rather than only using different coding rates and squared constellations to increase the performance of the system.]
Purpose	[Include non-square constellations in 802.15.3a standard.]
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Proposal

This paper provides a rational for the use of non-squared constellations in PAN systems rather than only using different coding rates and squared constellations to increase the performance of the system.

In the PAN receiver, there are inherent operating limits of the receiver (phase tracking, time recovery, equalization, etc...) that need a certain minimum signal to noise ratio to work. If we use a higher constellation by reducing the code rate even if the FEC system is able to make the signal to be decoded, because of the lower signal to noise ratio, the adaptive elements of the receiver may not work properly.

The steps between consecutive squared constellations are too big, especially for small constellations. An intermediate step (corresponding to odd powers of 2) will make the system to work with more granularity obtaining greater coverage for a determined data rate.

From our experience, the use of only squared constellations with different code rates (how it is done in 802.16) does not help to the system. The receiver cannot rely in decreasing the coding rate to make the system works because of the adaptive elements in the receiver (phase tracking, time recovery, equalization, etc...) which need a higher signal to noise ratio to work correctly. With the use of non-squared constellations, the receiver works with an appropriate signal to noise ratio thus providing a higher data rate.

In this paper, we propose a method for using non-square QAM constellations with independent I&Q. This method is based on the creation of non-separable I and Q constellations by combining constituent separable I and Q constellations.

Description Of The Method

The way to obtain this objective has the following steps:

1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points) (i.e. if we want to use a 32 QAM we start with the constellation for 64 QAM with independent I&Q Gray mapping).

2. Delete every other point in each dimension such that every row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation mapping, where one bit position of the I value or one bit position of the Q value have been removed. The first number is from the I dimension, called I-value, and the second number is from the Q dimension, called Q-value.

4. Decoded the resulting signal using two subset square constellations with independent I and Q and reducing the processing power because of factor that appear in the decoding process.

With this technique, the resulting non-square constellation can be decoded with independently.

When one bit in one dimension is removed, the resulting n-1 bits of the n bit constellation-value have independent I and Q.

Application of the method to the 2 QAM case.

The 2 QAM case is a special case where the two resulting points can always decoded independently.

Application of the method to the 8 QAM case.

The design of an 8 QAM constellation with independent I&Q property, using Gray mapping has 4 possible combinations or cases. The first 2 steps are common all 4 cases. Steps 3 and 4 are unique for each case.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 8 QAM, n=3, the square constellation is 16 QAM with independent I&Q Gray mapping.

Figure 1 shows the 16 QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
I     {0 {0 {1 {1
       0} 1} 1} 0}
                                    Q
      00 01 03 02    (00)
      10 11 13 12    (01)
      30 31 33 32    (11)
      20 21 23 22    (10)
```

Figure 1. 16 QAM constellation with Gray Mapping.

Step 2. Delete every other point in each dimension such that each row keeps half of the poi7nts and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 2 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 2.

```
   01    02
10    13
   31    32
20    23
```

Figure 2. Second step of the method.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 3 shows the region created in this case.

```
remove the most protected bit of I

I-value {}        I    {0}{1}{1}{0}
Q-value ()                             Q
                          01    00    (00)
                       10    11       (01)
                          31    30    (11)
                       20    21       (10)
```

Figure 3. 8 QAM case 1.

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 4.

```
   01----00
10--|-11  |
 | 31--|-30
20----21
```

Figure 4. subset constellation of 8 QAM case 1

Case 3.2. Remove the least protected bit in I. Figure 5 shows the region created in this case.

```
remove the least protected bit of I

I-value {}        I    {0}{0}{1}{1}
Q-value ()                             Q
                          00    01      (00)
                       10    11         (01)
                          30    31      (11)
                       20    21         (10)
```

Figure 5. 8 QAM case 2.

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 6.

```
   00----01
10--|-11  |
 | 30--|-31
20----21
```

Figure 6. subset constellation of 8 QAM case 2

In this case 2 it is important to note that the OX values of the two subset square constellations are the same 0 for the first column and 1 for the second column. This fact is very helpful to the decoding process, reducing considerably the computational requirements.

Case 3.3. Remove the most protected bit Q. Figure 7 shows the region created in this case.

```
remove the most protected bit of Q

I-value {}        I    {0 {1 {1 {0
Q-value ()              0} 0} 1} 1}            Q

                           02    01          (0)
                        10    13             (1)
                           12    11          (1)
                        00    03             (0)
```

Figure 7. 8 QAM case 3

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 8.

```
   02----01
10--|-13  |
 | 12--|-11
00----03
```

Figure 8. subset constellation of 8 QAM case 3

Case 3.4. Remove the least protected bit Q. Figure 9shows the region created in this case.

```
remove the least protected bit of Q

I-value {}        I    {0 {1 {1 {0
Q-value ()              0} 0} 1} 1}            Q

                           02    01          (0)
                        00    03             (0)
                           12    11          (1)
                        10    13             (1)
```

Figure 9. 8 QAM case 4.

Also in this case it is interesting to note that in the previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 10.

```
   02----01
00--|-03  |
 | 12--|-11
10----13
```

Figure 10. subset constellation of 8 QAM case 4

In this case 4 it is important to note that the OY values of the two subset square constellations are the same 0 for the first row and 1 for the second row. This fact is very helpful to the decoding process, reducing considerably the computational requirements.

Application of the method to the 32 QAM case.

Here we illustrate the use of the method for a 32 QAM constellation with Gray mapping for the 6 possible combinations.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 32 QAM, n=5, the square constellation is 64 QAM with independent I&Q Gray mapping.

Figure 11 shows the 64QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
0  0  0  0  1  1  1  1
0  0  1  1  1  1  0  0
0  1  1  0  0  1  1  0

00 01 03 02 06 07 05 04 (000)
10 11 13 12 16 17 15 14 (001)
30 31 33 32 36 37 35 34 (011)
20 21 23 22 26 27 25 24 (010)
60 61 63 62 66 67 65 64 (110)
70 71 73 72 76 77 75 74 (111)
50 51 53 52 56 57 55 54 (101)
40 41 43 42 46 47 45 44 (100)
```

Figure 11. 64 QAM constellation with Gray Mapping.

Step 2. Delete every other point in each dimension such that each row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 12 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 12.

```
   01    02    07    04
10    13    16    15
   31    32    37    34
20    23    26    25
   61    62    67    64
70    73    76    75
   51    52    57    54
40    43    46    45
```

Figure 12. Second step of the method for the 32 QAM case.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 13 shows the region created in this case.

```
remove the most protected bit of I

 I-value {}        I   {0 {0 {1 {1 {1 {1 {0 {0
 Q-value ()             0} 1} 1} 0} 0} 1} 1} 0}         Q

                           01    02    03    00       (000)
                        10    13    12    11          (001)
                           31    32    33    30       (011)
                        20    23    22    21          (010)
                           61    62    63    60       (110)
                        70    73    72    71          (111)
                           51    52    53    50       (101)
                        40    43    42    41          (100)
```

Figure 13. 32 QAM case 1.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 14.

```
   01----02----03----00
10--|-13----12----11  |
 | 31    32    33  | 30
20  | 23    22    21  |
 | 61    62    63  | 60
70  | 73    72    71  |
 | 51----52----53--|-50
40----43----42----41
```

Figure 14. subset constellation of 32 QAM case 1

The other cases:
Case 3.2. Remove the second most protected bit in I.
Case 3.3. Remove the least bit in I.
Case 3.4. Remove most protected bit in Q.
Case 3.5. Remove the second most protected bit in Q.
Case 3.6. Remove the least protected bit in Q.
Are done in the same way.

Application of the method to the 128 QAM case.

In this point we illustrate the use of the method for a 128 QAM constellation with Gray mapping for the 16 possible combinations.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 128 QAM, n=7, the square constellation is 256 QAM with independent I&Q Gray mapping.

Figure 15 shows the 256 QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1
0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0
0  1  1  0  0  1  1  0  0  1  1  0  0  1  1  0

00 01 03 02 06 07 05 04 0C 0D 0F 0E 0A 0B 09 08 (0000)
10 11 13 12 16 17 15 14 1C 1D 1F 1E 1A 1B 19 18 (0001)
30 31 33 32 36 37 35 34 3C 3D 3F 3E 3A 3B 39 38 (0011)
20 21 23 22 26 27 25 24 2C 2D 2F 2E 2A 2B 29 28 (0010)
60 61 63 62 66 67 65 64 6C 6D 6F 6E 6A 6B 69 68 (0110)
70 71 73 72 76 77 75 74 7C 7D 7F 7E 7A 7B 79 78 (0111)
50 51 53 52 56 57 55 54 5C 5D 5F 5E 5A 5B 59 58 (0101)
40 41 43 42 46 47 45 44 4C 4D 4F 4E 4A 4B 49 48 (0100)
C0 C1 C3 C2 C6 C7 C5 C4 CC CD CF CE CA CB C9 C8 (1100)
D0 D1 D3 D2 D6 D7 D5 D4 DC DD DF DE DA DB D9 D8 (1101)
F0 F1 F3 F2 F6 F7 F5 F4 FC FD FF FE FA FB F9 F8 (1111)
E0 E1 E3 E2 E6 E7 E5 E4 EC ED EF EE EA EB E9 E8 (1110)
A0 A1 A3 A2 A6 A7 A5 A4 AC AD AF AE AA AB A9 A8 (1010)
B0 B1 B3 B2 B6 B7 B5 B4 BC BD BF BE BA BB B9 B8 (1011)
90 91 93 92 96 97 95 94 9C 9D 9F 9E 9A 9B 99 98 (1001)
80 81 83 82 86 87 85 84 8C 8D 8F 8E 8A 8B 89 88 (1000)
```

Figure 15. 256 QAM constellation with Gray Mapping.

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Step 2. Delete every other point in each dimension such that each row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 16 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 16.

```
     01    02    07    04    0D    0E    0B    08
10    13    16    15    1C    1F    1A    19
     31    32    37    34    3D    3E    3B    38
20    23    26    25    2C    2F    2A    29
     61    62    67    64    6D    6E    6B    68
70    73    76    75    7C    7F    7A    79
     51    52    57    54    5D    5E    5B    58
40    43    46    45    4C    4F    4A    49
     C1    C2    C7    C4    CD    CE    CB    C8
D0    D3    D6    D5    DC    DF    DA    D9
     F1    F2    F7    F4    FD    FE    FB    F8
E0    E3    E6    E5    EC    EF    EA    E9
     A1    A2    A7    A4    AD    AE    AB    A8
B0    B3    B6    B5    BC    BF    BA    B9
     91    92    97    94    9D    9E    9B    98
80    83    86    85    8C    8F    8A    89
```

Figure 16. Second step of the method for the 128 QAM case.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 17 shows the region created in this case.

remove the most protected bit of I

```
I-value {}I   {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1 {0 {0 {0 {0
Q-value ()     0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0           Q
               0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    05    06    03    00        (0000)
              10    13    16    15    14    17    12    11           (0001)
                 31    32    37    34    35    36    33    30        (0011)
              20    23    26    25    24    27    22    21           (0010)
                 61    62    67    64    65    66    63    60        (0110)
              70    73    76    75    74    77    72    71           (0111)
                 51    52    57    54    55    56    53    50        (0101)
              40    43    46    45    44    47    42    41           (0100)
                 C1    C2    C7    C4    C5    C6    C3    C0        (1100)
              D0    D3    D6    D5    D4    D7    D2    D1           (1101)
                 F1    F2    F7    F4    F5    F6    F3    F0        (1111)
              E0    E3    E6    E5    E4    E7    E2    E1           (1110)
                 A1    A2    A7    A4    A5    A6    A3    A0        (1010)
              B0    B3    B6    B5    B4    B7    B2    B1           (1011)
                 91    92    97    94    95    96    93    90        (1001)
              80    83    86    85    84    87    82    81           (1000)
```

Figure 17. 128 QAM case 1.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 18.

```
   01----02----07----04----05----06----03----00
10--|-13----16----15----14----17----12----11  |
 | 31    32    37    34    35    36    33   | 30
20  | 23    26    25    24    27    22    21  |
 | 61    62    67    64    65    66    63   | 60
70  | 73    76    75    74    77    72    71  |
 | 51    52    57    54    55    56    53   | 50
40  | 43    46    45    44    47    42    41  |
 | C1    C2    C7    C4    C5    C6    C3   | C0
D0  | D3    D6    D5    D4    D7    D2    D1  |
 | F1    F2    F7    F4    F5    F6    F3   | F0
E0  | E3    E6    E5    E4    E7    E2    E1  |
 | A1    A2    A7    A4    A5    A6    A3   | A0
B0  | B3    B6    B5    B4    B7    B2    B1  |
 | 91----92----97----94----95----96----93--|-90
80----83----86----85----84----87----82----81
```

Figure 18. subset constellation of 128 QAM case 1.

The other cases:

Case 3.2. Remove the second most protected bit in I.
Case 3.3. Remove the third most protected bit in I.
Case 3.4. Remove the least protected bit in I.
Case 3.5. Remove the most protected bit in Q.
Case 3.6. Remove second most protected bit in Q.
Case 3.7. Remove the third most protected bit in Q.
Case 3.8. Remove the least protected bit in Q.

Are done in the same way.

Higher order modulations

The same idea could be applied to higher order constellations.

Conclusion

We propose a way to use non-squared constellations that uses constituent independent I&Q constellations because of the computational complexity advantages that the use of these constellations provide.