

CONTRIBUTION

TITLE: Performance Results of the method provided in T1E1.4/2000-183
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ABSTRACT

A previous contribution T1E1.4/2000-217 presents performance results of the method described in T1E1.4/2000-183. These performance results presented in T1E1.4/200-217 are not totally correct. In this contribution we present the performance results of the technique described in T1E1.3/2000-183 as well as the performance results according to the ITU requirements provided in the ITU document BA-108R1.

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1. Introduction

A previous contribution T1E1.4/2000-217 presents performance results of the method described in T1E1.4/2000-183. The performance results presented in T1E1.4/200-217 are for an interleaver size of only 1024 bits. It is more realistic to use interleaver sizes between 4000 and 30000 bits. In this contribution we present the performance results of the technique described in T1E1.3/2000-183 as well as the performance results according to the ITU requirements provided in BA-108R1

The performance of the method described in T1E1.4/2000-183 for an interleaver size of 1024 bits is presented in table 1.

Table 1. Performance of T1E1.4/2000-183 using an interleaver size of 1024.

Spectral Efficiency [bit/s/Hz]	# of error bit [bits]	minimum SNR (Capacity) [dB]	SNR Required (Coded) [dB]	SNR Required (uncoded) [dB]
1	1	0	2.1	11.64
2	2	4.77	7.5	15.09
3	1	8.45	10.9	18.78
4	2	11.76	13.9	21.83
6	2	17.99	21.8	27.79
8	2	24.07	28.9	33.87
10	2	30.10	36.3	39.91
12	2	35.30	42.2	46.50

If a more realistic interleaver, as such of 10500 bits is used, the performance results are presented in table 2.

Table 2. Performance of T1E1.4/2000-183 using an interleaver size of 10500.

Spectral Efficiency [bit/s/Hz]	# of error bit [bits]	minimum SNR (Capacity) [dB]	SNR Required (Coded) [dB]	SNR Required (uncoded) [dB]
1	1	0	2.0	11.64
2	2	4.77	7.01	15.09
3	1	8.45	10.2	18.78
4	2	11.76	13.4	21.83
6	2	17.99	20.8	27.79
8	2	24.07	27.5	33.87
10	2	30.10	33.8	39.91
12	2	35.30	39.2	46.50

2. Mapping

The method presented in T1E1.4/2000-183 uses a special Gray mapping. This Gray mapping protects more the information bits than the parity bits. This concept is shown in figure 1 for the case 64 QAM with a spectral efficiency of 2 bit/s/Hz (2 information bits and 1 parity bit).

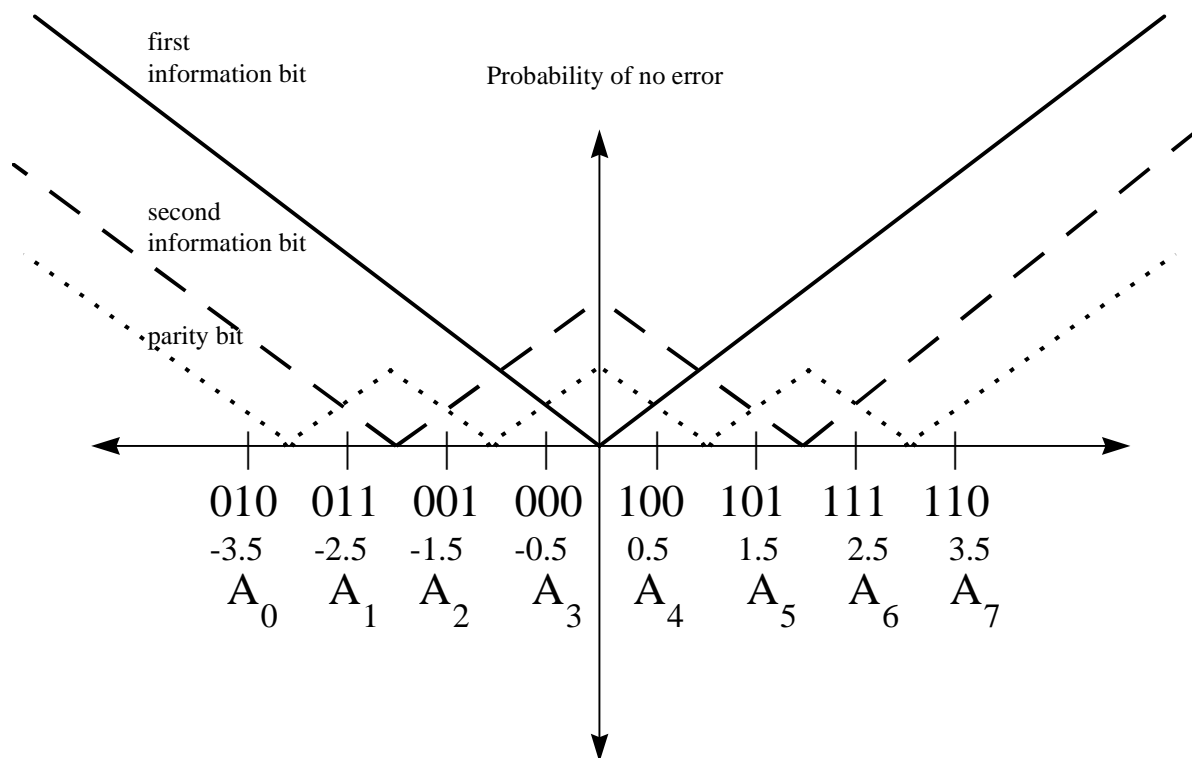


Figure 1. Mapping of the 64 QAM proposed in T1E1.4/2000-183

The fact that the information bits are more protected than the parity bits provides an additional coding gain around 0.5 dB.

The waterfall curves of the method proposed in T1E1.4/2000-183 are very steep. For this reason, in terms of BER it is translated in almost two orders of magnitude improvement.

For a given E_b/N_0 if the information bits are more protected the BER is $1.8 \cdot 10^{-7}$ and if the parity bits are more protected the BER is $7.1 \cdot 10^{-5}$.

3. Description of the method for its implementation

3.1 Capacity Bounds

The minimum E_b/N_0 values to achieve the Shannon bound 64 QAM and 16384 QAM bounds for spectral efficiencies of 4 and 12 bits/s/Hz respectively are as in Table 3 for a BER= 10^{-5} .

Table 3. Shannon bounds.

Spectral efficiency η [bit/s/Hz]	Shannon bound E_b/N_0 [dB]
4	5.6
12	24.7

The conversion from SNR to E_b/N_0 is performed using the following relation

$$E_b/N_0[\text{dB}] = \text{SNR} [\text{dB}] - 10 \log_{10}(\eta) [\text{dB}] \tag{1}$$

where η is the number of information bits per symbol.

For a D-dimension modulation the following formulae are used:

$$\text{SNR} = \frac{E[|a_k^2|]}{E[|w_k^2|]} = \frac{E[|a_k^2|]}{D\sigma_N^2} = \frac{E_{av}}{D\sigma_N^2} \tag{2}$$

$$\text{SNR} = \frac{E_s}{D\frac{N_0}{2}} = \frac{\eta E_b}{D\frac{N_0}{2}} \tag{3}$$

where σ_N^2 is the noise variance in each of the D dimension and η is the number of information bits per symbol. From the above relations:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} \tag{4}$$

3.2 Coding

The proposed coding scheme is shown in Figure 1. The two systematic recursive codes (SRC) used are identical and are defined in Figure 2. The code is described by the generating polynomials 35o and 23o.

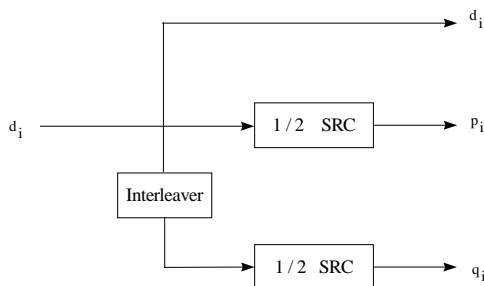


Figure 1

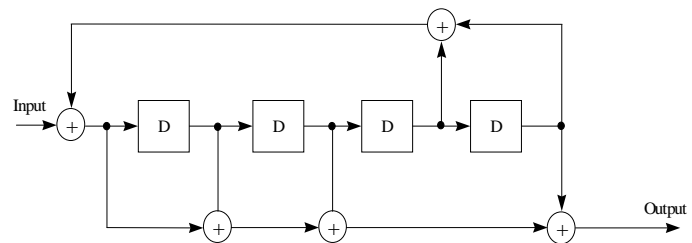


Figure 2

3.3 Turbo code internal interleaver.

The Turbo code internal interleaver consists of bits-input to a rectangular matrix, intra-row and inter-row permutations of the rectangular matrix, and bits-output from the rectangular matrix with pruning. The bits input to the Turbo code internal interleaver are denoted by $x_1, x_2, x_3, \dots, x_K$, where K is the integer number of the bits and takes one value of $40 \leq K \leq 32000$. The relation between the bits input to the Turbo code internal interleaver and the bits input to the channel coding is defined by $x_k = o_{irk}$ and $K = K_i$.

K	Number of bits input to Turbo code internal interleaver
R	Number of rows of rectangular matrix
C	Number of columns of rectangular matrix
p	Prime number
v	Primitive root
s(i)	Base sequence for intra-row permutation
qj	Minimum prime integers
rj	Permuted prime integers
T(j)	Inter-row permutation pattern
Uj(i)	Intra-row permutation pattern
i	Index of matrix
j	Index of matrix
k	Index of bit sequence

3.3.1 Bits-input to rectangular matrix

The bit sequence input to the Turbo code internal interleaver x_k is written into the rectangular matrix as follows.

- (1) Determine the number of rows R of the rectangular matrix such that:

$$R = \begin{cases} 5, & \text{if } (40 \leq K \leq 159) \\ 10, & \text{if } ((160 \leq K \leq 200) \text{ or } (481 \leq K \leq 530)) \\ 20, & \text{if } (K = \text{any other value}) \end{cases}$$

where the rows of rectangular matrix are numbered 0, 1, 2, ..., $R - 1$ from top to bottom.

- (2) Determine the number of columns C of rectangular matrix such that:

if $(481 \leq K \leq 530)$ then

$$p = 53 \text{ and } C = p.$$

else

Find minimum prime p such that

$$(p + 1) - K/R \geq 0,$$

and determine C such that

if $(p - K/R \geq 0)$ then

if $(p - 1 - K/R \geq 0)$ then

$$C = p - 1.$$

else

$$C = p.$$

end if

else

$$C = p + 1$$

end if

end if

where the columns of rectangular matrix are numbered 0, 1, 2, ..., $C - 1$ from left to right.

(3) Write the input bit sequence x_k into the $R \times C$ rectangular matrix row by row starting with bit x_1 in column 0 of row 0:

$$\begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_C \\ x_{(C+1)} & x_{(C+2)} & x_{(C+3)} & \cdots & x_{2C} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{((R-1)C+1)} & x_{((R-1)C+2)} & x_{((R-1)C+3)} & \cdots & x_{RC} \end{bmatrix}$$

3.3.2 Intra-row and inter-row permutations

After the bits-input to the $R \times C$ rectangular matrix, the intra-row and inter-row permutations for the $R \times C$ rectangular matrix are performed by using the following algorithm.

- (1) Select a primitive root v from table 4.
- (2) Construct the base sequence $s(i)$ for intra-row permutation as:

$$s(i) = [v \times s(i - 1)] \bmod p, \quad i = 1, 2, \dots, (p - 2), \text{ and } s(0) = 1.$$

- (3) Let $q_0 = 1$ be the first prime integer in $\{q_j\}$, and select the consecutive minimum prime integers $\{q_j\}$ ($j = 1, 2, \dots, R - 1$) such that:

$$\text{g.c.d}\{q_j, p - 1\} = 1, \quad q_j > 6, \text{ and } q_j > q_{(j-1)},$$

where g.c.d. is greatest common divisor.

- (4) Permute $\{q_j\}$ to make $\{r_j\}$ such that

$$r_{T(j)} = q_j, \quad j = 0, 1, \dots, R - 1,$$

where $T(j)$ ($j = 0, 1, 2, \dots, R - 1$) is the inter-row permutation pattern defined as the one of the

following four kind of patterns: Pat_1 , Pat_2 , Pat_3 and Pat_4 depending on the number of input bits K .

$$\{T(0), T(1), T(2), \dots, T(R-1)\} = \begin{cases} Pat_4 & \text{if } (40 \leq K \leq 159) \\ Pat_3 & \text{if } (160 \leq K \leq 200) \\ Pat_1 & \text{if } (201 \leq K \leq 480) \\ Pat_3 & \text{if } (481 \leq K \leq 530) \\ Pat_1 & \text{if } (531 \leq K \leq 2280) \\ Pat_2 & \text{if } (2281 \leq K \leq 2480) \\ Pat_1 & \text{if } (2481 \leq K \leq 3160) \\ Pat_2 & \text{if } (3161 \leq K \leq 3210) \\ Pat_1 & \text{if } (3211 \leq K) \end{cases},$$

where Pat_1 , Pat_2 , Pat_3 and Pat_4 have the following patterns respectively.

$Pat_1: \{19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 10, 8, 13, 17, 3, 1, 16, 6, 15, 11\}$

$Pat_2: \{19, 9, 14, 4, 0, 2, 5, 7, 12, 18, 16, 13, 17, 15, 3, 1, 6, 11, 8, 10\}$

$Pat_3: \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$

$Pat_4: \{4, 3, 2, 1, 0\}$

(5) Perform the j -th ($j = 0, 1, 2, \dots, R - 1$) intra-row permutation as:

if ($C = p$) then

$$U_j(i) = s([i \times r_j] \bmod (p - 1)), \quad i = 0, 1, 2, \dots, (p - 2), \text{ and } U_j(p - 1) = 0,$$

where $U_j(i)$ is the input bit position of i -th output after the permutation of j -th row.

end if

if ($C = p + 1$) then

$$U_j(i) = s([i \times r_j] \bmod (p - 1)), \quad i = 0, 1, 2, \dots, (p - 2), \text{ and } U_j(p - 1) = 0, \text{ and } U_j(p) = p,$$

where $U_j(i)$ is the input bit position of i -th output after the permutation of j -th row, and

if ($K = C \times R$) then

Exchange $U_{R-1}(p)$ with $U_{R-1}(0)$.

end if

end if

if ($C = p - 1$) then

$$U_j(i) = s([i \times r_j] \bmod (p - 1)) - 1, \quad i = 0, 1, 2, \dots, (p - 2),$$

where $U_j(i)$ is the input bit position of i -th output after the permutation of j -th row.

end if

(6) Perform the inter-row permutation based on the pattern $T(j)$ ($j = 0, 1, 2, \dots, R - 1$), where $T(j)$ is the original row position of the j -th permuted row.

Table 4: Table of prime p and associated primitive root v

p	v	p	v	p	v	p	v	p	v
7	3	313	10	709	2	1129	11	1597	11
11	2	317	2	719	11	1151	17	1601	3
13	2	331	3	727	5	1153	5	1607	5
17	3	337	10	733	6	1163	5	1609	7
19	2	347	2	739	3	1171	2	1613	3
23	5	349	2	743	5	1181	7	1619	2
29	2	353	3	751	3	1187	2	1621	2
31	3	359	7	757	2	1193	3	1627	3
37	2	367	6	761	6	1201	11	1637	2
41	6	373	2	769	11	1213	2	1657	11
43	3	379	2	773	2	1217	3	1663	3
47	5	383	5	787	2	1223	5	1667	2
53	2	389	2	797	2	1229	2	1669	2
59	2	397	5	809	3	1231	3	1693	2
61	2	401	3	811	3	1237	2	1697	3
67	2	409	21	821	2	1249	7	1699	3
71	7	419	2	823	3	1259	2	1709	3
73	5	421	2	827	2	1277	2	1721	3
79	3	431	7	829	2	1279	3	1723	3
83	2	433	5	839	11	1283	2	1733	2
89	3	439	15	853	2	1289	6	1741	2
97	5	443	2	857	3	1291	2	1747	2
101	2	449	3	859	2	1297	10	1753	7
103	5	457	13	863	5	1301	2	1759	6
107	2	461	2	877	2	1303	6	1777	5
109	6	463	3	881	3	1307	2	1783	10
113	3	467	2	883	2	1319	13	1787	2
127	3	479	13	887	5	1321	13	1789	6
131	2	487	3	907	2	1327	3	1801	11
137	3	491	2	911	17	1361	3	1811	6
139	2	499	7	919	7	1367	5	1823	5
149	2	503	5	929	3	1373	2	1831	3
151	6	509	2	937	5	1381	2	1847	5
157	5	521	3	941	2	1399	13	1861	2
163	2	523	2	947	2	1409	3	1867	2
167	5	541	2	953	3	1423	3	1871	14
173	2	547	2	967	5	1427	2	1873	10
179	2	557	2	971	6	1429	6	1877	2
181	2	563	2	977	3	1433	3	1879	6
191	19	569	3	983	5	1439	7	1889	3
193	5	571	3	991	6	1447	3	1901	2
197	2	577	5	997	7	1451	2	1907	2
199	3	587	2	1009	11	1453	2	1913	3
211	2	593	3	1013	3	1459	3	1931	2
223	3	599	7	1019	2	1471	6	1933	5
227	2	601	7	1021	10	1481	3	1949	2
229	6	607	3	1031	14	1483	2	1951	3
233	3	613	2	1033	5	1487	5	1973	2
239	7	617	3	1039	3	1489	14	1979	2
241	7	619	2	1049	3	1493	2	1987	2
251	6	631	3	1051	7	1499	2	1993	5
257	3	641	3	1061	2	1511	11	1997	2
263	5	643	11	1063	3	1523	2	1999	3
269	2	647	5	1069	6	1531	2		
271	6	653	2	1087	3	1543	5		
277	5	659	2	1091	2	1549	2		
281	3	661	2	1093	5	1553	3		
283	3	673	5	1097	3	1559	19		
293	2	677	2	1103	5	1567	3		
307	5	683	5	1109	2	1571	2		
311	17	691	3	1117	2	1579	3		

3.3.3 Bits-output from rectangular matrix with pruning

After intra-row and inter-row permutations, the bits of the permuted rectangular matrix are denoted by y'_k :

$$\begin{bmatrix} y'_1 & y'_{(R+1)} & y'_{(2R+1)} & \cdots & y'_{((C-1)R+1)} \\ y'_2 & y'_{(R+2)} & y'_{(2R+2)} & \cdots & y'_{((C-1)R+2)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ y'_R & y'_{2R} & y'_{3R} & \cdots & y'_{CR} \end{bmatrix}$$

The output of the Turbo code internal interleaver is the bit sequence read out column by column from the intra-row and inter-row permuted $R \times C$ matrix starting with bit y'_1 in row 0 of column 0 and ending with bit y'_{CR} in row $R - 1$ of column $C - 1$. The output is pruned by deleting bits that were not present in the input bit sequence, i.e. bits y'_k that corresponds to bits x_k with $k > K$ are removed from the output. The bits output from Turbo code internal interleaver are denoted by x'_1, x'_2, \dots, x'_K , where x'_1 corresponds to the bit y'_k with smallest index k after pruning, x'_2 to the bit y'_k with second smallest index k after pruning, and so on. The number of bits output from Turbo code internal interleaver is K and the total number of pruned bits is: $R \times C - K$.

3.4. Coding And Modulation For 4 Bit/S/Hz Spectral Efficiency.

3.4.1 Puncturing

In order to obtain a rate 4/6 code, the puncturing pattern used is shown in Table 5.

Table 5. Puncturing and Mapping for Rate 4/6 64 QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄
parity bit (p)	p ₁	-	-	-
parity bit (q)	-	-	q ₃	-
8AM symbol (I)	(d ₁ , d ₂ , p ₁)			
8AM symbol (Q)	(d ₃ , d ₄ , q ₃)			
64 QAM symbol (I, Q)	(I, Q)=(d ₁ , d ₂ , p ₁ , d ₃ , d ₄ , q ₃)			

3.4.2 Modulation

Gray mapping was used in each dimension. Four information bits are required to be sent using a 64 QAM constellation. For a rate 4/6 code and 64 QAM, the noise variance in each dimension is

$$E_{av} = (8(49+25+9+1+25+49+49+9+49+1+25+9+25+1+9+1)) A^2 / 64 = 42 A^2 \quad (10)$$

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_0} \right)^{-1} = 42 A^2 \left(\frac{2 \times 4 \times E_b}{N_0} \right)^{-1} = 5.25 A^2 \left(\frac{E_b}{N_0} \right)^{-1} \quad (11)$$

The puncturing and mapping scheme is shown in Table 5 for 4 consecutive information bits that are encoded into 6 coded bits, therefore one 64 QAM symbol. The turbo encoder with the puncturing presented in Table 5 is a rate 4/6 turbo code which in conjunction with 64 QAM gives a spectral efficiency of 4 bits/s/Hz. Considering two independent Gaussian noises with identical variance σ_N^2 , the LLR can be determined independently for each I and Q. It is assumed that at time k u_1^k, u_2^k and u_3^k modulate the I component and u_4^k, u_5^k and u_6^k modulate the Q component of the 64 QAM scheme. At

the receiver, the I and Q signals are treated independently in order to take advantage of the simpler formulae for the LLR values.

3.4.3 Bit Probabilities

The 8AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k)$, where u_1^k is the most significant bit and u_3^k is the least significant bit . The following set can be defined.

1. bit-1-is-1 = { A₄, A₅, A₆ , A₇ }
2. bit-2-is-1 = { A₀, A₁ , A₆ , A₇ }
3. bit-3-is-1 = { A₁ ,A₂ , A₅ , A₆ }

From each received symbol, the bit probabilities are computed as follows:

$$LLR(u_n^k) = \log \left(\frac{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2} (I^k - d_{1i,n}^k)^2]}{\sum_{i=1}^4 \exp[-\frac{I}{2\sigma_N^2} (I^k - d_{0i,n}^k)^2]} \right) \tag{12}$$

For I dimension. An identical computation effort is required for the Q dimension, the I^k being replaced with the Q^k demodulated value in order to evaluate LLR(u₄^k), LLR(u₅^k) and LLR(u₆^k).

3.4.4 Simulation Results

Figure 3 shows the simulation results for 10,400 information bits with interleaver option 1. A BER of 10⁻⁷ can be achieved after 8 iterations at E_b/N₀ = 8.3 dB.

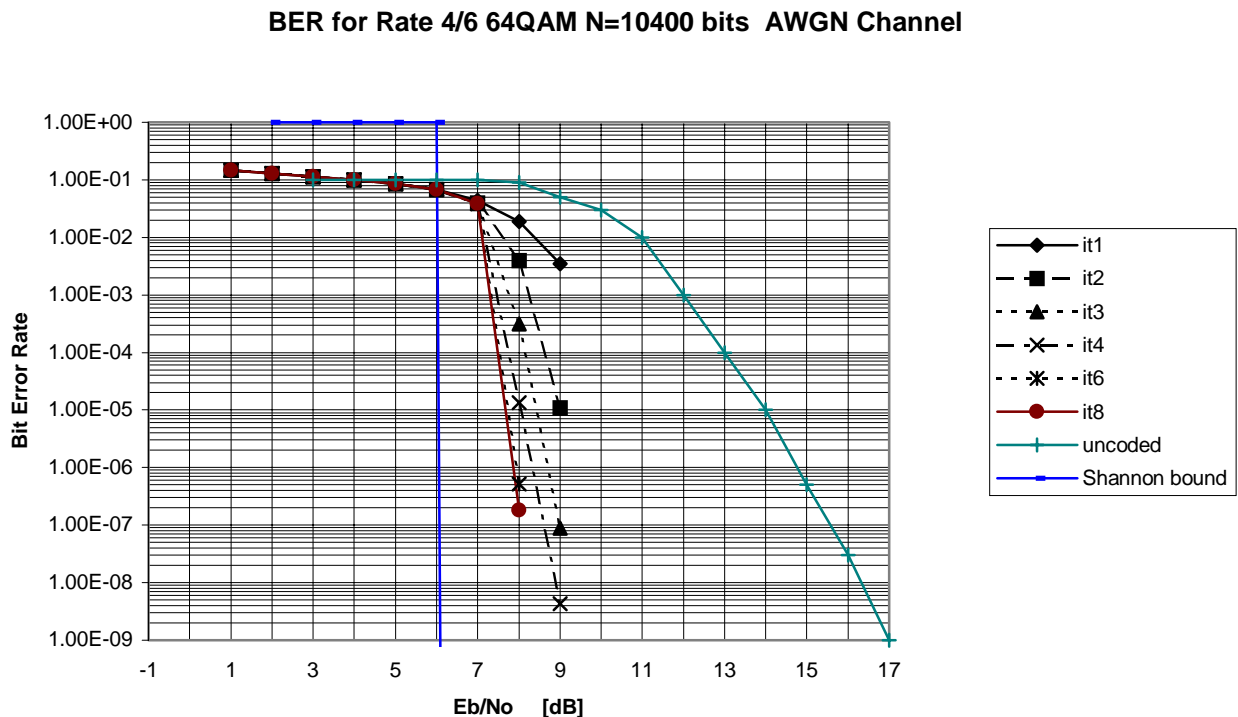


Figure 3.

3.5 Coding And Modulation For 12 Bit/S/Hz Spectral Efficiency.

This section investigated a rate 12/14 coding scheme with 16384 QAM.

3.5.1 Puncturing

In order to obtain a rate 12/14 code, the puncturing pattern used is shown in Table 6.

Table 6. Puncturing and Mapping for Rate 12/14 16384 QAM

Information bit (d)	d ₁	d ₂	d ₃	d ₄	d ₅	d ₆	d ₇	d ₈	d ₉	d ₁₀	d ₁₁	d ₁₂
parity bit (p)	p ₁	-	-	-	-	-	-	-	-	-	-	-
parity bit (q)	-	-	-	-	-	-	q ₇	-	-	-	-	-
128AM symbol (I)	(d ₁ , d ₂ , d ₃ , d ₄ , d ₅ , d ₆ , p ₁)											
128AM symbol (Q)	(d ₇ , d ₈ , d ₉ , d ₁₀ , d ₁₁ , d ₁₂ , q ₇)											
16384 QAM symbol (I, Q)	(d ₁ , d ₂ , d ₃ , d ₄ , d ₅ , d ₆ , p ₁ , d ₇ , d ₈ , d ₉ , d ₁₀ , d ₁₁ , d ₁₂ , q ₇)											

3.5.2 Modulation

For a 16384 QAM constellation with points at -127A, -125A, -123A, -121A, -119A, -117A, -115A, -113A, -111A, -109A, -107A, -105A, -103A, -101A, -99A, -97A, -95A, -93A, -91A, -89A, -87A, -85A, -83A, -81A, -79A, -77A, -75A, -73A, -71A, -69A, -67A, -65A, -63A, -61A, -59A, -57A, -55A, -53A, -51A, -49A, -47A, -45A, -43A, -41A, -39A, -37A, -35A, -33A, -31A, -29A, -27A, -25A, -23A, -21A, -19A, -17A, -15A, -13A, -11A, -9A, -7A, -5A, -3A, -A, A, 3A, 5A, 7A, 9A, 11A, 13A, 15A, 17A, 19A, 21A, 23A, 25A, 27A, 29A, 31A, 33A, 35A, 37A, 39A, 41A, 43A, 45A, 47A, 49A, 51A, 53A, 55A, 57A, 59A, 61A, 63A, 65A, 67A, 69A, 71A, 73A, 75A, 77A, 79A, 81A, 83A, 95A, 87A, 89A, 91A, 93A, 95A, 97A, 99A, 101A, 103A, 105A, 107A, 109A, 111A, 113A, 115A, 117A, 119A, 121A, 123A, 125A, 127A. E_{av} is:

$$E_{av} = 5461 A^2 \quad (13)$$

It is assumed that at time k the symbol $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k, u_8^k, u_9^k, u_{10}^k, u_{11}^k, u_{12}^k, u_{13}^k, u_{14}^k)$ is sent though the channel. It is assumed that at time k the symbol $u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k$ and u_7^k modulate the I component and $u_8^k, u_9^k, u_{10}^k, u_{11}^k, u_{12}^k, u_{13}^k$ and u_{14}^k modulate the Q component of a 16384 QAM scheme.

For a rate 12/14 code and 16384 QAM, the noise variance is:

$$\sigma_N^2 = E_{av} \left(\frac{2\eta E_b}{N_o} \right)^{-1} = 5461 A^2 \left(\frac{2 \times 6 \times E_b}{N_o} \right)^{-1} = 455.08 A^2 \left(\frac{E_b}{N_o} \right)^{-1} \quad (14)$$

In order to study the performance of this scheme, a rate 6/7 turbo code and a 128AM is used. The 16384 QAM scheme will achieve a similar performance in terms of bit error rate (BER) at twice the spectral efficiency, assuming an ideal demodulator. The puncturing and mapping scheme shown in Table 6 is for 12 consecutive information bits that are coded into 14 encoded bits, therefore, one 16384 QAM symbol. The turbo encoder is a rate 12/14 turbo code, which in conjunction with 16384 QAM, gives a spectral efficiency of 12 bits/s/Hz.

3.5.3 Bit Probabilities

The 128AM symbol is defined as $u^k = (u_1^k, u_2^k, u_3^k, u_4^k, u_5^k, u_6^k, u_7^k)$, where u_1^k is the most significant bit and u_7^k is the least significant bit. The following set can be defined.

1. bit-1-is-1 = { A₆₄, A₆₅, A₆₆, A₆₇, A₆₈, A₆₉, A₇₀, A₇₁, A₇₂, A₇₃, A₇₄, A₇₅, A₇₆, A₇₇, A₇₈, A₇₉, A₈₀, A₈₁, A₈₂, A₈₃, A₈₄, A₈₅, A₈₆, A₈₇, A₈₈, A₈₉, A₉₀, A₉₁, A₉₂, A₉₃, A₉₄, A₉₅, A₉₆, A₉₇, A₉₈, A₉₉, A₁₀₀, A₁₀₁, A₁₀₂, A₁₀₃, A₁₀₄, A₁₀₅, A₁₀₆, A₁₀₇, A₁₀₈, A₁₀₉, A₁₁₀, A₁₁₁, A₁₁₂, A₁₁₃, A₁₁₄, A₁₁₅, A₁₁₆, A₁₁₇, A₁₁₈, A₁₁₉, A₁₂₀, A₁₂₁, A₁₂₂, A₁₂₃, A₁₂₄, A₁₂₅, A₁₂₆, A₁₂₇ }
2. bit-2-is-1 = { A₃₂, A₃₃, A₃₄, A₃₅, A₃₆, A₃₇, A₃₈, A₃₉, A₄₀, A₄₁, A₄₂, A₄₃, A₄₄, A₄₅, A₄₆, A₄₇, A₄₈, A₄₉, A₅₀, A₅₁, A₅₂, A₅₃, A₅₄, A₅₅, A₅₆, A₅₇, A₅₈, A₅₉, A₆₀, A₆₁, A₆₂, A₆₃, A₆₄, A₆₅, A₆₆, A₆₇, A₆₈, A₆₉, A₇₀, A₇₁, A₇₂, A₇₃, A₇₄, A₇₅, A₇₆, A₇₇, A₇₈, A₇₉, A₈₀, A₈₁, A₈₂, A₈₃, A₈₄, A₈₅, A₈₆, A₈₇, A₈₈, A₈₉, A₉₀, A₉₁, A₉₂, A₉₃, A₉₄, A₉₅ }
3. bit-3-is-1 = { A₁₆, A₁₇, A₁₈, A₁₉, A₂₀, A₂₁, A₂₂, A₂₃, A₂₄, A₂₅, A₂₆, A₂₇, A₂₈, A₂₉, A₃₀, A₃₁, A₃₂, A₃₃, A₃₄, A₃₅, A₃₆, A₃₇, A₃₈, A₃₉, A₄₀, A₄₁, A₄₂, A₄₃, A₄₄, A₄₅, A₄₆, A₄₇, A₈₀, A₈₁, A₈₂, A₈₃, A₈₄, A₈₅, A₈₆, A₈₇, A₈₈, A₈₉, A₉₀, A₉₁, A₉₂, A₉₃, A₉₄, A₉₅, A₉₆, A₉₇, A₉₈, A₉₉, A₁₀₀, A₁₀₁, A₁₀₂, A₁₀₃, A₁₀₄, A₁₀₅, A₁₀₆, A₁₀₇, A₁₀₈, A₁₀₉, A₁₁₀, A₁₁₁ }
4. bit-4-is-1 = { A₈, A₉, A₁₀, A₁₁, A₁₂, A₁₃, A₁₄, A₁₅, A₁₆, A₁₇, A₁₈, A₁₉, A₂₀, A₂₁, A₂₂, A₂₃, A₄₀, A₄₁, A₄₂, A₄₃, A₄₄, A₄₅, A₄₆, A₄₇, A₄₈, A₄₉, A₅₀, A₅₁, A₅₂, A₅₃, A₅₄, A₅₅, A₇₂, A₇₃, A₇₄, A₇₅, A₇₆, A₇₇, A₇₈, A₇₉, A₈₀, A₈₁, A₈₂, A₈₃, A₈₄, A₈₅, A₈₆, A₈₇, A₁₀₄, A₁₀₅, A₁₀₆, A₁₀₇, A₁₀₈, A₁₀₉, A₁₁₀, A₁₁₁, A₁₁₂, A₁₁₃, A₁₁₄, A₁₁₅, A₁₁₆, A₁₁₇, A₁₁₈, A₁₁₉, }
5. bit-5-is-1 = { A₄, A₅, A₆, A₇, A₈, A₉, A₁₀, A₁₁, A₂₀, A₂₁, A₂₂, A₂₃, A₂₄, A₂₅, A₂₆, A₂₇, A₃₆, A₃₇, A₃₈, A₃₉, A₄₀, A₄₁, A₄₂, A₄₃, A₅₂, A₅₃, A₅₄, A₅₅, A₅₆, A₅₇, A₅₈, A₅₉, A₆₈, A₆₉, A₇₀, A₇₁, A₇₂, A₇₃, A₇₄, A₇₅, A₈₄, A₈₅, A₈₆, A₈₇, A₈₈, A₈₉, A₉₀, A₉₁, A₁₀₀, A₁₀₁, A₁₀₂, A₁₀₃, A₁₀₄, A₁₀₅, A₁₀₆, A₁₀₇, A₁₁₆, A₁₁₇, A₁₁₈, A₁₁₉, A₁₂₀, A₁₂₁, A₁₂₂, A₁₂₃, }
6. bit-6-is-1 = { A₀, A₁, A₆, A₇, A₈, A₉, A₁₄, A₁₅, A₁₆, A₁₇, A₂₂, A₂₃, A₂₄, A₂₅, A₃₀, A₃₁, A₃₂, A₃₃, A₃₈, A₃₉, A₄₀, A₄₁, A₄₆, A₄₇, A₄₈, A₄₉, A₅₄, A₅₅, A₅₆, A₅₇, A₆₂, A₆₃, A₆₄, A₆₅, A₇₀, A₇₁, A₇₂, A₇₃, A₇₈, A₇₉, A₈₀, A₈₁, A₈₆, A₈₇, A₈₈, A₈₉, A₉₄, A₉₅, A₉₆, A₉₇, A₁₀₂, A₁₀₃, A₁₀₄, A₁₀₅, A₁₁₀, A₁₁₁, A₁₁₂, A₁₁₃, A₁₁₈, A₁₁₉, A₁₂₀, A₁₂₁, A₁₂₆, A₁₂₇ }
7. bit-7-is-1 = { A₁, A₂, A₅, A₆, A₉, A₁₀, A₁₃, A₁₄, A₁₇, A₁₈, A₂₁, A₂₂, A₂₅, A₂₆, A₂₉, A₃₀, A₃₃, A₃₄, A₃₇, A₃₈, A₄₁, A₄₂, A₄₅, A₄₆, A₄₉, A₅₀, A₅₃, A₅₄, A₅₇, A₅₈, A₆₁, A₆₂, A₆₅, A₆₆, A₆₉, A₇₀, A₇₃, A₇₄, A₇₇, A₇₈, A₈₁, A₈₂, A₈₅, A₈₆, A₈₉, A₉₀, A₉₃, A₉₄, A₉₇, A₉₈, A₁₀₁, A₁₀₂, A₁₀₅, A₁₀₆, A₁₀₉, A₁₁₀, A₁₁₃, A₁₁₄, A₁₁₇, A₁₁₈, A₁₂₁, A₁₂₂, A₁₂₅, A₁₂₆ }

From each received symbol, R^k , the bit probabilities are computed as follows:

$$LLR(u_n^k) = \log \left(\frac{\sum_{A_i \in \text{bit-}n\text{-is-}1} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_i\|\right)}{\sum_{A_j \in \text{bit-}n\text{-is-}0} \exp\left(-\frac{1}{2\sigma_N^2} \|R^k - A_j\|\right)} \right) \quad (17)$$

2.5.4 Simulation Results

Figure 4 shows the simulation results for 31200 information bits with interleaver option 1. A BER of 10^{-7} can be achieved after 8 iterations at $E_b/N_0 = 28.25$ dB.

BER for Rate 12/14 16384QAM N=31200 bits AWGN Channel

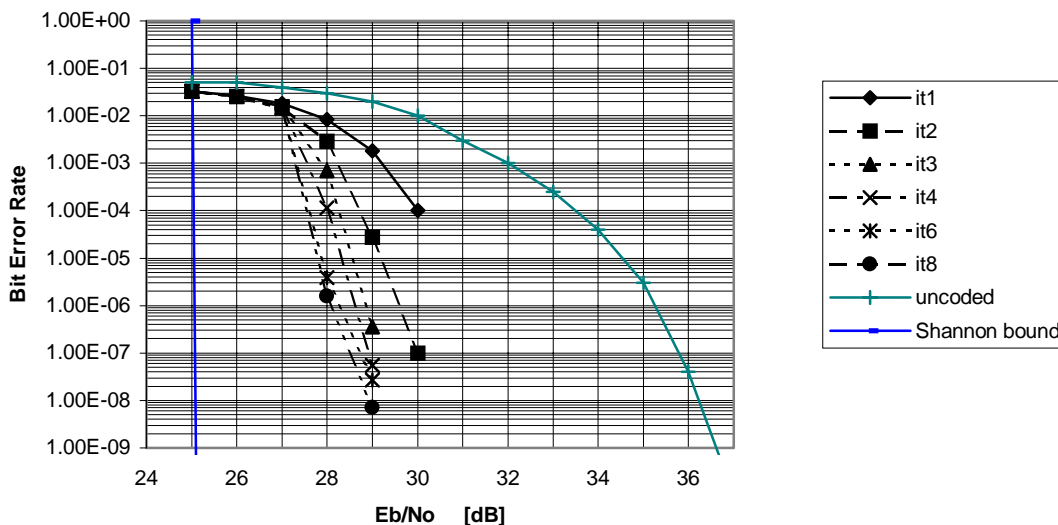


Figure 4

3. Results.

3.1 Without Reed-Solomon

3.1.1 Net Coding Gain.

Table 7. Net Coding Gain

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	4.60	7.42	7.94
		800	2	1.5	3.70	4.92	4.84
		400	1	0.7	3.30	3.62	3.84
	200	10,400	13	10.0	4.60	7.52	8.14
		1,600	2	1.5	4.10	6.42	6.64
		800	1	0.7	3.70	4.92	4.84
12	100	15,600	13	10.0	4.10	5.91	6.03
		2,400	2	1.5	3.60	5.51	5.63
		1,200	1	0.7	3.00	3.91	4.03
	200	31,200	13	10.0	4.10	6.81	7.53
		4,800	2	1.5	3.60	5.91	6.43
		2,400	1	0.7	3.60	5.51	5.63

3.1.2 Errors due to Impulse noise.

The impulse noise is defined as 2 consecutive DMT symbols with an increase AWGN respect to the reference noise level of a carrier-to-noise ratio of 21.5 dB (for 4 bit/tone) and 45.5 dB (for 12 bit/tone).

Table 8. Error due to Impulse Noise

Bit/Tone	Tones	Interleaver Size	RL + 2.5 dB	RL + 5 dB	RL + 7.5 dB	RL + 10 dB	RL + 12.5 dB	RL + 15 dB	RL + 17.5 dB	RL + 20 dB
4	100	5,200	0	0	0	0	0	0	0	4
		800	0	0	39	65	104	140	188	243
		400	0	0	10	50	89	127	161	214
	200	10,400	0	0	0	0	0	0	0	7
		1,600	0	0	0	127	189	267	363	448
		800	0	0	40	116	187	252	346	440
12	100	15,600	0	0	0	0	10	58	130	207
		2,400	0	0	40	78	121	171	216	295
		1,200	0	0	43	98	129	188	255	329
	200	31,200	0	0	0	0	90	175	313	482
		4,800	0	0	75	177	254	341	462	608
		2,400	0	0	80	166	244	345	457	598

3.1.3 Error Statistics.

3.1.3.1 For AWGN

Table 9. Error Statistics for AWGN

Bit/Tone	Tones	Interleaver Size	1 consec. error	2 consec. errors	3 consec. errors	4 consec. errors	5 consec. errors	6 consec. errors
4	100	5,200	87.30%	10.81%	1.47%	0.29%	0.03%	0.10%
		800	94.35%	5.64%	0.00%	0.00%	0.00%	0.00%
		400	90.28%	9.72%	0.01%	0.00%	0.00%	0.00%
	200	10,400	89.90%	8.63%	1.21%	0.20%	0.06%	0.00%
		1,600	97.94%	2.06%	0.00%	0.00%	0.00%	0.00%
		800	90.28%	9.72%	0.01%	0.00%	0.00%	0.00%
12	100	15,600	99.79%	0.21%	0.00%	0.00%	0.00%	0.00%
		2,400	98.72%	1.28%	0.00%	0.00%	0.00%	0.00%
		1,200	97.94%	2.06%	0.00%	0.00%	0.00%	0.00%
	200	31,200	99.86%	0.14%	0.00%	0.00%	0.00%	0.00%
		4,800	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		2,400	98.72%	1.28%	0.00%	0.00%	0.00%	0.00%

3.1.3.2 Impulse Noise

Table 10. Error Statistics for Impulse noise

Bit/Tone	Tones	Interleaver Size	1 consec. error	2 consec errors	3 consec errors	4 consec errors	5 consec errors	6 consec errors
4	100	5,200	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		800	75.97%	18.99%	18.99%	3.36%	0.84%	0.84%
		400	79.89%	17.24%	1.72%	1.15%	0.00%	0.00%
	200	10,400	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		1,600	80.24%	13.05%	4.47%	1.68%	0.47%	0.00%
		800	79.03%	17.50%	2.46%	0.46%	0.46%	0.00%
12	100	15,600	95.19%	4.81%	0.00%	0.00%	0.00%	0.00%
		2,400	94.61%	5.28%	0.11%	0.00%	0.00%	0.00%
		1,200	93.63%	5.95%	93.63%	0.00%	0.00%	0.00%
	200	31,200	93.25%	6.65%	0.00%	0.00%	0.00%	0.00%
		4,800	94.89%	4.95%	0.16%	0.00%	0.00%	0.00%
		2,400	94.59%	5.36%	0.06%	0.00%	0.00%	0.00%

It is interesting that for the large turbo decoders the impulse errors still tends to stay within the 2 DMT symbols. This implies a moderately large turbo coder of 5 ms follow by a convolutional interleaver/Reed Solomon of 10 ms should create both robust performance and good impulse resistance.

3.2 With Reed-Solomon

3.2.1 Net Coding Gain.

Table 11. Coding Gain

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	5.00	8.62	9.64
		800	2	1.5	3.50	7.12	8.44
		400	1	0.7	3.50	6.42	7.44
	200	10,400	13	10.0	5.30	8.82	9.84
		1,600	2	1.5	4.60	7.72	8.74
		800	1	0.7	3.50	7.12	8.44
12	100	15,600	13	10.0	4.40	7.71	8.53
		2,400	2	1.5	4.60	7.41	8.33
		1,200	1	0.7	4.10	6.81	7.63
	200	31,200	13	10.0	4.40	7.71	8.53
		4,800	2	1.5	4.40	7.21	8.13
		2,400	1	0.7	4.60	7.41	8.33

Table 12. Net Coding Gain

Bit/Tone	Tones	Interleaver Size	# of DMT symbols	Latency (Tx+Rx) ms <	10^{-3}	10^{-7}	10^{-9} extrap.
4	100	5,200	13	10.0	3.42	7.04	8.06
		800	2	1.5	1.78	5.40	6.72
		400	1	0.7	1.94	4.86	5.88
	200	10,400	13	10.0	3.72	7.24	8.26
		1,600	2	1.5	2.88	6.00	7.02
		800	1	0.7	1.94	5.56	6.88
12	100	15,600	13	10.0	0.20	3.51	4.33
		2,400	2	1.5	0.02	2.83	3.75
		1,200	1	0.7	-0.24	2.47	3.29
	200	31,200	13	10.0	1.06	4.37	5.19
		4,800	2	1.5	0.76	3.57	4.49
		2,400	1	0.7	0.26	3.07	3.99

3.2.2 Errors due to Impulse noise.

The impulse noise is defined as 2 consecutive DMT symbols with an increase AWGN respect to the reference noise level of a carrier-to-noise ratio of 21.5 dB (for 4 bit/tone) and 45.5 dB (for 12 bit/tone).

Table 13. Error due to Impulse Noise

Bit/Tone	Tones	Interleaver Size	RL + 2.5 dB	RL + 5 dB	RL + 7.5 dB	RL + 10 dB	RL + 12.5 dB	RL + 15 dB	RL + 17.5 dB	RL + 20 dB
4	100	5,200	0	0	0	0	0	0	0	0
		800	0	0	0	0	0	0	0	0
		400	0	0	0	0	0	0	0	0
	200	10,400	0	0	0	0	0	0	0	0
		1,600	0	0	0	0	0	0	0	0
		800	0	0	0	0	0	0	0	0
12	100	15,600	0	0	0	0	10	58	130	207
		2,400	0	0	0	0	0	0	0	0
		1,200	0	0	0	0	0	0	0	0
	200	31,200	0	0	0	0	90	175	313	482
		4,800	0	0	0	0	0	10	11	65
		2,400	0	0	0	0	9	10	24	115

3.2.3 Error Statistics.

The statistics results obtained are practically the same than for the non Reed-Solomon case