ITU - Telecommunication Standardization Sector Temporary Document **CF-038**

STUDY GROUP 15 Original: English

Clearwater, Florida, 08 - 12 January 2001

Question: 4/15

SOURCE[1]: VOCAL Technologies Ltd. (http://www.vocal.com)

TITLE: G.gen: G.vdsl: G.dmt.bis: G.lite.bis: : Method for using non-squared QAM constellations with independent I&Q for Receiver soft-Decision Decoding Techniques.

.

ABSTRACT

This paper proposes a method for using non-square QAM constellations with independent I&Q. This method is based in the creation of non-separable I and Q constellations by combining constituent separable I and Q constellations. This technique is applicable to any Receiver soft-Decision Decoding Technique, such as, Turbo codes or LDPC codes.

[1] Contact: Frederic Hirzel E: fhirzel@vocal.com
Juan Alberto Torres, Ph. D. E: jatorres@vocal.com
Victor Demjanenko, Ph. D. E: victord@vocal.com
VOCAL Technologies, Ltd. T: +1(716) 688 4675
Buffalo, NY 14228, USA F: +1(716) 639 0713

1. Introduction

This paper proposes a method for using non-square QAM constellations with independent I&Q. This method is based in the creation of non-separable I and Q constellations by combining constituent separable I and Q constellations. This technique is applicable to any Receiver soft-Decision Decoding Technique, such as Turbo codes or LDPC codes.

2. Independent I & Q constellations

The conventional technique for extraction soft-decision information from the channel is create a value representing the probability of the received symbol being a one as:

$$\frac{\sum \textit{of the measures with the transmit symbol was 0}}{\sum \textit{of the measures with the transmit symbol was 1}} \tag{1}$$

where the measure is defined as:

$$e^{(-n*metric)} \quad ; \quad n = \frac{1}{No}, \frac{1}{2No} \tag{2}$$

and where:

metric = Euclidian distance (or square of the Euclidian distance) from the possible transmit symbol to the received symbol.

For example, for the 16 point constellation:

```
(-3,+3) (-1,+3) (+1,+3) (+3,+3)
(-3,+1) (-1,+1) (+1,+1) (+3,+1)
(-3,-1) (-1,-1) (+1,-1) (+3,-1)
(-3,-3) (-1,-3) (+1,-3) (+3,-3)
```

With the symbols assignments of:

```
0000    0001    0011    0010
0100    0101    0111    0110
1100    1101    1111    1110
1000    1001    1011    1010
```

Using a two-digit representation:

```
I      {0 {0 {1 {1
        0} 1} 1} 0}
                     Q
       00 01 03 02  (00)
       10 11 13 12  (01)
       30 31 33 32  (11)
       20 21 23 22  (10)
```

and the point assignments of:

```
p30     p31     p32     p33
p20     p21     p22     P23
p10     p11     p12     p13
p00     p01     p02     p03
```

$$p_{i,j} = \left(x_{i,j}, y_{i,j}\right) \quad (3)$$

In order to extract the probability of the least significant bit for received value q = (u,v), the transmit points would be separated into those points whose least significant bit would be 1 and those points whose least significant bit would 0 as:

```
bit == 1    p31, p21, p11, p01
            p32, p22, p12, p02

bit == 0    p30, p20, p10, p00
            p33, P23, p13, p03
```

and the metrics mij would be:

$$m_{i,j} = \| p_{i,j} - q \|^{2} = \left(x_{i,j} - u \right)^2 + \left(y_{i,j} - v \right)^2 \quad (4)$$

and the metrics in respect to the least significant bit would be:

```
bit == 1    m31, m21, m11, m01
            m32, m22, m12, m02

bit == 0    m30, m20, m10, m00
            m33, m23, m13, m03
```

The sum of the measures in respect to the least significant bit and the value extracted from the channel would be:

$$value = \frac{S1}{S0} = \frac{\sum_{bit=1} e^{(-n^{*} m_{ij})} \quad ij = 31, 21, 11, 01, 32, 22, 12, 02}{\sum_{bit=0} e^{-(n^{*} m_{ij})} \quad ij = 30, 20, 10, 00, 33, 23, 13, 03} \quad (5)$$

For this example, there are 16 possible transmit symbols and there are also 16 calculation needed for creating the value extracted from the channel.

The example was selected to also illustrate a constellation constructed and bits assigned such that the soft-decision information extraction has reduced complexity due to both the constellation's shape and the independent dimension assignments of bits the constellation's symbols.

For S1, the summation of the measures to each transmit symbol whose bit is 1 is given as:

$$S1 = \sum_{bit=1} e^{(-n^{*} m_{ij})} \qquad ij = 31,21,11,01,32,22,12,02 \quad (6)$$

This summations can be separated into two summations, each for a "column" of possible constellations values as:

$$S1 = S11 + S12 \quad (7)$$

where:

$$S11 = \sum_{bit=1} e^{(-n^* m_{ij})} \qquad ij = 31,21,11,01 \tag{8}$$

$$S12 = \sum_{bit=1} e^{(-n^* m_{ij})} \qquad ij = 32,22,12,02 \tag{9}$$

since mij is defined as:

$$m_{i\,j} = \left(x_{i\,j} - u\ \right)^2 + \left(\ y_{i\,j} - v\ \right)^2 = m\ x_{i\,j} + m\ y_{i\,j} \tag{10}$$

where:

$$mx_{i\,j} = \left(x_{i\,j} - u\ \right)^2 \tag{11}$$

$$my_{i\,j} = \left(y_{i\,j} - v\ \right)^2 \tag{12}$$

and using the property,

$$u^{(x+y)} = u^x\, u^y \tag{13}$$

It is easily shown the S11, S12 are, for this constellation and bit assignment, accepting the notations:

$$mx_j = \left(column\ \ j - u\ \right)^2 \tag{14}$$

$$my_i = \left(row\, i - v\ \right)^2 \tag{15}$$

$$S11 = Sy\ e^{(-n^* mx_j)} \qquad j = 1 \quad choose\ any\ value\ \ for\ i \tag{16}$$

$$S12 = Sy\ e^{(-n^* mx_j)} \qquad j = 2 \quad choose\ any\ value\ \ for\ i \tag{17}$$

$$Sy = \sum e^{(-n^* my_i)} \qquad i = 0,1,2,3 \quad choose\ any\ value\ \ for\ \ j \tag{18}$$

and thus, S1 can be defined as:

$$S1\ =\ Sy\ \ Sx1 \quad ; \quad Sx1\ = \sum e^{(-n^* mx_j)} \qquad j\ =\ 1\ \ ,2 \tag{19}$$

and similarly, S0 can be defined as:

$$S0\ =\ Sy\ \ Sx0 \quad ; \quad Sx0 = \sum e^{(-n^* mx_j)} \qquad j\ =\ 0,3 \tag{20}$$

and the ratio S1/S0 becomes:

$$value\ =\ \frac{S1}{S0}\ =\ \frac{Sy\ \ Sx1}{Sy\ \ Sx0} \tag{21}$$

$$value\ =\ \frac{Sx1}{Sx0}\ =\ \frac{\sum e^{(-n\ mx_j)}\ \ ; j = 1,2}{\sum e^{(-n\ mx_j)}\ \ ; j = 0,3} \tag{22}$$

which requires only 4 calculations instead of 16 calculations.

Of course, the same reduction of calculations will occur for all bits.

This technique for reducing the processing complexity is frequently described as creating constellation with separable I and Q dimensions.

N even

$N=2^n$ QAM	#exp	#add	#mul	#div	TOTAL
Full	$2N^{1/2}$	$n(N-2)$	N	n	$2N^{1/2}+(n+1)N-n$
Independent I&Q	$2N^{1/2}$	$n(N^{1/2}-2)$	0	n	$(n+2)N^{1/2}+n$

The increase complexity for this type of constellation can be shown to be of $O((N)^{1/2})$ where N is the number of constellation points.

3. Description Of The Method

The way to obtain this objective has the following steps:

1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points) (i.e. if we want to use a 32 QAM we start with the constellation for 64 QAM with independent I&Q Gray mapping).

2. Delete every other point in each dimension such that every row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation mapping, where one bit position of the I value or one bit position of the Q value have been removed. The first number is from the I dimension, called I-value, and the second number is from the Q dimension, called Q-value.

4. Decoded the resulting signal using two subset square constellations with independent I and Q and reducing the processing power because of factor that appear in the decoding process.

With this technique, the resulting non-square constellation can be decoded with independent probabilities.

When one bit in one dimension is removed, the resulting n-1 bits of the n bit constellation-value have independent I and Q.

3.1 2 QAM case.

The 2 QAM case is a special case where the two resulting points can always decoded independently.

3.2 Application of the method to the 8 QAM case.

The design of an 8 QAM constellation with independent I&Q property, using Gray mapping has 4 possible combinations or cases. The first 2 steps are common all 4 cases. Steps 3 and 4 are unique for each case.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 8 QAM, n=3, the square constellation is 16 QAM with independent I&Q Gray mapping.

Figure 1 shows the 16 QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
I  {0 {0 {1 {1
    0} 1} 1} 0}
                   Q
   00 01 03 02    (00)
   10 11 13 12    (01)
   30 31 33 32    (11)
   20 21 23 22    (10)
```

Figure 1. 16 QAM constellation with Gray Mapping.

Step 2. Delete every other point in each dimension such that each row keeps half of the poi7nts and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 2 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 2.

```
   01    02
10    13
   31    32
20    23
```

Figure 2. Second step of the method.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 3 shows the region created in this case.

```
remove the most protected bit of I

I-value {}      I    {0}{1}{1}{0}
Q-value ()                             Q
                        01    00      (00)
                     10    11         (01)
                        31    30      (11)
                     20    21         (10)
```

Figure 3. 8 QAM case 1.

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 4.

```
   01----00
10--|-11  |
 | 31--|-30
20----21
```

Figure 4. subset constellation of 8 QAM case 1

Case 3.2. Remove the least protected bit in I. Figure 5 shows the region created in this case.

```
remove the least protected bit of I

I-value {}      I    {0}{0}{1}{1}
Q-value ()                            Q
                        00    01     (00)
                     10    11        (01)
                        30    31     (11)
                     20    21        (10)
```

Figure 5. 8 QAM case 2.

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 6.

```
   00----01
10--|-11  |
 | 30--|-31
20----21
```

Figure 6. subset constellation of 8 QAM case 2

In this case 2 it is important to note that the OX values of the two subset square constellations are the same 0 for the first column and 1 for the second column. This fact is very helpful to the decoding process, reducing considerably the computational requirements.

Case 3.3. Remove the most protected bit Q. Figure 7 shows the region created in this case.

```
remove the most protected bit of Q

I-value {}      I    {0 {1 {1 {0
Q-value ()           0} 0} 1} 1}          Q

                        02    01         (0)
                     10    13            (1)
                        12    11         (1)
                     00    03            (0)
```

Figure 7. 8 QAM case 3

The previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 8.

```
   02----01
10--|-13  |
 | 12--|-11
00----03
```

Figure 8. subset constellation of 8 QAM case 3

Case 3.4. Remove the least protected bit Q. Figure 9shows the region created in this case.

```
remove the least protected bit of Q

I-value {}      I    {0 {1 {1 {0
Q-value ()           0} 0} 1} 1}          Q

                        02    01         (0)
                     00    03            (0)
                        12    11         (1)
                     10    13            (1)
```

Figure 9. 8 QAM case 4.

Also in this case it is interesting to note that in the previous case the resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 10.

```
   02----01
00--|-03  |
 | 12--|-11
10----13
```

Figure 10. subset constellation of 8 QAM case 4

In this case 4 it is important to note that the OY values of the two subset square constellations are the same 0 for the first row and 1 for the second row. This fact is very helpful to the decoding process, reducing considerably the computational requirements.

3.3 Application of the method to the 32 QAM case.

Here we illustrate the use of the method for a 32 QAM constellation with Gray mapping for the 6 possible combinations.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 32 QAM, n=5, the square constellation is 64 QAM with independent I&Q Gray mapping.

Figure 11 shows the 64QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
0  0  0  0  1  1  1  1
0  0  1  1  1  1  0  0
0  1  1  0  0  1  1  0

00 01 03 02 06 07 05 04 (000)
10 11 13 12 16 17 15 14 (001)
30 31 33 32 36 37 35 34 (011)
20 21 23 22 26 27 25 24 (010)
60 61 63 62 66 67 65 64 (110)
70 71 73 72 76 77 75 74 (111)
50 51 53 52 56 57 55 54 (101)
40 41 43 42 46 47 45 44 (100)
```

Figure 11. 64 QAM constellation with Gray Mapping.

Step 2. Delete every other point in each dimension such that each row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 12 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 12.

```
   01    02    07    04
10    13    16    15
   31    32    37    34
20    23    26    25
   61    62    67    64
70    73    76    75
   51    52    57    54
40    43    46    45
```

Figure 12. Second step of the method for the 32 QAM case.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 13 shows the region created in this case.

```
remove the most protected bit of I

I-value {}      I   {0 {0 {1 {1 {1 {1 {0 {0
Q-value ()           0} 1} 1} 0} 0} 1} 1} 0}       Q

                        01    02    03    00     (000)
                     10    13    12    11        (001)
                        31    32    33    30     (011)
                     20    23    22    21        (010)
                        61    62    63    60     (110)
                     70    73    72    71        (111)
                        51    52    53    50     (101)
                     40    43    42    41        (100)
```

Figure 13. 32 QAM case 1.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 14.

```
   01----02----03----00
10--|-13----12----11  |
 | 31    32    33   | 30
20  | 23    22    21  |
 | 61    62    63   | 60
70  | 73    72    71  |
 | 51----52----53--|-50
40----43----42----41
```

Figure 14. subset constellation of 32 QAM case 1

Case 3.2. Remove the second most protected bit in I. Figure 15 shows the region created in this case.

```
remove the second most protected bit of I

I-value {}      I   {0 {0 {0 {0 {1 {1 {1 {1
Q-value ()           0} 1} 1} 0} 0} 1} 1} 0}       Q

                        01    00    03    02     (000)
                     10    11    12    13        (001)
                        31    30    33    32     (011)
                     20    21    22    23        (010)
                        61    60    63    62     (110)
                     70    71    72    73        (111)
                        51    50    53    52     (101)
                     40    41    42    43        (100)
```

Figure 15. 32 QAM case 2.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 16.

```
   01----00----03----02
10--|-11----12----13  |
 | 31    30    33  | 32
20  | 21    22    23  |
 | 61    60    63  | 62
70  | 71    72    73  |
 | 51----50----53--|-52
40----41----42----43
```

Figure 16. subset constellation of 32 QAM case 2.

Case 3.3. Remove the least bit in I. Figure 17 shows the region created in this case.

```
remove the least protected bit of I

I-value {}      I   {0 {0 {0 {0 {1 {1 {1 {1
Q-value ()           0} 0} 1} 1} 1} 1} 0} 0}       Q

                        00    01    03    02     (000)
                     10    11    13    12        (001)
                        30    31    33    32     (011)
                     20    21    23    22        (010)
                        60    61    63    62     (110)
                     70    71    73    72        (111)
                        50    51    53    52     (101)
                     40    41    43    42        (100)
```

Figure 17. 32 QAM case 3.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 18.

```
   00----01----03----02
10--|-11----13----12  |
 | 30    31    33  | 32
20  | 21    23    22  |
 | 60    61    63  | 62
70  | 71    73    72  |
 | 50----51----53--|-52
40----41----43----42
```

Figure 18. subset constellation of 32 QAM case 3.

Case 3.4. Remove most protected bit in Q. Figure 19 shows the region created in this case.

```
remove the most protected bit of Q

I-value {}      I   {0 {0 {0 {0 {1 {1 {1 {1
Q-value ()           0  0  1  1  1  1  0  0
                     0} 1} 1} 0} 0} 1} 1} 0}      Q

                        01    02    07    04      (00)
                     10    13    16    15         (01)
                        31    32    37    34      (11)
                     20    23    26    25         (10)
                        21    22    27    24      (10)
                     30    33    36    35         (11)
                        11    12    17    14      (01)
                     00    03    06    05         (00)
```

Figure 19. 32 QAM case 4.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 20.

```
   01----02----07----04
10--|-13----16----15  |
 | 31    32    37  | 34
20  | 23    26    25  |
 | 21    22    27  | 24
30  | 33    36    35  |
 | 11----12----17--|-14
00----03----06----05
```

Figure 20. subset constellation of 32 QAM case 4.

Case 3.5. Remove the second most protected bit in Q. Figure 21 shows the region created in this case.

```
remove the second most protected bit of Q

I-value {}      I   {0 {0 {0 {0 {1 {1 {1 {1
Q-value ()           0  0  1  1  1  1  0  0
                     0} 1} 1} 0} 0} 1} 1} 0}        Q

                        01    02    07    04      (00)
                     10    13    16    15         (01)
                        11    12    17    14      (01)
                     00    03    06    05         (00)
                        21    22    27    24      (10)
                     30    33    36    35         (11)
                        31    32    37    34      (11)
                     20    23    26    25         (10)
```

Figure 21. 32 QAM case 5.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 22.

```
   01----02----07----04
10--|-13----16----15  |
 | 11    12    17  | 14
00  | 03    06    05  |
 | 21    22    27  | 24
30  | 33    36    35  |
 | 31----32----37--|-34
20----23----26----25
```

Figure 22. subset constellation of 32 QAM case 5.

Case 3.6. Remove the least protected bit in Q. Figure 23 shows the region created in this case.

```
remove the least protected bit of Q

I-value {}      I   {0 {0 {0 {0 {1 {1 {1 {1
Q-value ()           0  0  1  1  1  1  0  0
                     0} 1} 1} 0} 0} 1} 1} 0}       Q

                        01    02    07    04      (00)
                     00    03    06    05         (00)
                        11    12    17    14      (01)
                     10    13    16    15         (01)
                        31    32    37    34      (11)
                     30    33    36    35         (11)
                        21    22    27    24      (10)
                     20    23    26    25         (10)
```

Figure 23. 32 QAM case 6.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 24.

```
   01----02----07----04
00--|-03----06----05  |
 | 11    12    17  | 14
10  | 13    16    15  |
 | 31    32    37  | 34
30  | 33    36    35  |
 | 21----22----27--|-24
20----23----26----25
```

Figure 24. subset constellation of 32 QAM case 5.

3.4 Application of the method to the 128 QAM case.

In this point we illustrate the use of the method for a 128 QAM constellation with Gray mapping for the 16 possible combinations.

Step1. Draw the square constellation that has double number of points (2^{n+1} points) than the non-square constellation that we want to use (2^n points). For 128 QAM, n=7, the square constellation is 256 QAM with independent I&Q Gray mapping.

Figure 25 shows the 256 QAM constellation with Gray Mapping. The first number represent the Q dimension and the second number represents the I dimension.

```
0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1
0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0
0  1  1  0  0  1  1  0  0  1  1  0  0  1  1  0

00 01 03 02 06 07 05 04 0C 0D 0F 0E 0A 0B 09 08 (0000)
10 11 13 12 16 17 15 14 1C 1D 1F 1E 1A 1B 19 18 (0001)
30 31 33 32 36 37 35 34 3C 3D 3F 3E 3A 3B 39 38 (0011)
20 21 23 22 26 27 25 24 2C 2D 2F 2E 2A 2B 29 28 (0010)
60 61 63 62 66 67 65 64 6C 6D 6F 6E 6A 6B 69 68 (0110)
70 71 73 72 76 77 75 74 7C 7D 7F 7E 7A 7B 79 78 (0111)
50 51 53 52 56 57 55 54 5C 5D 5F 5E 5A 5B 59 58 (0101)
40 41 43 42 46 47 45 44 4C 4D 4F 4E 4A 4B 49 48 (0100)
C0 C1 C3 C2 C6 C7 C5 C4 CC CD CF CE CA CB C9 C8 (1100)
D0 D1 D3 D2 D6 D7 D5 D4 DC DD DF DE DA DB D9 D8 (1101)
F0 F1 F3 F2 F6 F7 F5 F4 FC FD FF FE FA FB F9 F8 (1111)
E0 E1 E3 E2 E6 E7 E5 E4 EC ED EF EE EA EB E9 E8 (1110)
A0 A1 A3 A2 A6 A7 A5 A4 AC AD AF AE AA AB A9 A8 (1010)
B0 B1 B3 B2 B6 B7 B5 B4 BC BD BF BE BA BB B9 B8 (1011)
90 91 93 92 96 97 95 94 9C 9D 9F 9E 9A 9B 99 98 (1001)
80 81 83 82 86 87 85 84 8C 8D 8F 8E 8A 8B 89 88 (1000)
```

Figure 25. 256 QAM constellation with Gray Mapping.

Step 2. Delete every other point in each dimension such that each row keeps half of the points and every column keeps half of the points and that the constellation points that remains have the same distance between them.

Figure 26 shows the constellation after removing half of the points.

The technique produces similar constellations if we decide to keep the points that had been removed from Figure 26.

```
   01    02    07    04    0D    0E    0B    08
10    13    16    15    1C    1F    1A    19
   31    32    37    34    3D    3E    3B    38
20    23    26    25    2C    2F    2A    29
   61    62    67    64    6D    6E    6B    68
70    73    76    75    7C    7F    7A    79
   51    52    57    54    5D    5E    5B    58
40    43    46    45    4C    4F    4A    49
   C1    C2    C7    C4    CD    CE    CB    C8
D0    D3    D6    D5    DC    DF    DA    D9
   F1    F2    F7    F4    FD    FE    FB    F8
E0    E3    E6    E5    EC    EF    EA    E9
   A1    A2    A7    A4    AD    AE    AB    A8
B0    B3    B6    B5    BC    BF    BA    B9
   91    92    97    94    9D    9E    9B    98
80    83    86    85    8C    8F    8A    89
```

Figure 26. Second step of the method for the 128 QAM case.

Step 3. Assign to each remaining point, a number formed from the bits of the I value and bits of the Q value of the original constellation map, where one bit position of the I value or one bit position of the Q value have been removed.

Case 3.1. Remove the most protected bit in I. Figure 27 shows the region created in this case.

remove the most protected bit of I

```
I-value {}I   {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1 {0 {0 {0 {0
Q-value ()     0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0         Q
               0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    05    06    03    00     (0000)
              10    13    16    15   14     17     12     11      (0001)
                 31    32    37    34    35    36    33    30     (0011)
              20    23    26    25   24     27     22     21      (0010)
                 61    62    67    64    65    66    63    60     (0110)
              70    73    76    75    74    77    72    71        (0111)
                 51    52    57    54    55    56    53    50     (0101)
              40    43    46    45    44    47    42    41        (0100)
                 C1    C2    C7    C4    C5    C6    C3    C0     (1100)
              D0    D3    D6    D5    D4    D7    D2    D1        (1101)
                 F1    F2    F7    F4    F5    F6    F3    F0     (1111)
              E0    E3    E6    E5    E4    E7    E2    E1        (1110)
                 A1    A2    A7    A4    A5    A6    A3    A0     (1010)
              B0    B3    B6    B5    B4    B7    B2    B1        (1011)
                 91    92    97    94    95    96    93    90     (1001)
              80    83    86    85    84    87    82    81        (1000)
```

Figure 27. 128 QAM case 1.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 28.

```
                 01----02----07----04----05----06----03----00
              10--|-13----16----15----14----17----12----11  |
               | 31    32    37    34    35    36    33 | 30
              20  | 23    26    25    24    27    22    21  |
               | 61    62    67    64    65    66    63 | 60
              70  | 73    76    75    74    77    72    71  |
               | 51    52    57    54    55    56    53 | 50
              40  | 43    46    45    44    47    42    41  |
               | C1    C2    C7    C4    C5    C6    C3 | C0
              D0  | D3    D6    D5    D4    D7    D2    D1  |
               | F1    F2    F7    F4    F5    F6    F3 | F0
              E0  | E3    E6    E5    E4    E7    E2    E1  |
               | A1    A2    A7    A4    A5    A6    A3 | A0
              B0  | B3    B6    B5    B4    B7    B2    B1  |
               | 91----92----97----94----95----96----93--|-90
              80----83----86----85----84----87----82----81
```

Figure 28. subset constellation of 128 QAM case 1.

Case 3.2. Remove the second most protected bit in I. Figure 29 shows the region created in this case.

remove the second most protected bit of I

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0        Q
               0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    03    00    05    06    07    04      (0000)
              10    13    12    11    14    17    16    15         (0001)
                 31    32    33    30    35    36    37    34      (0011)
              20    23    22    21    24    27    26    25         (0010)
                 61    62    63    60    65    66    67    64      (0110)
              70    73    72    71    74    77    76    75         (0111)
                 51    52    53    50    55    56    57    54      (0101)
              40    43    42    41    44    47    46    45         (0100)
                 C1    C2    C3    C0    C5    C6    C7    C4      (1100)
              D0    D3    D2    D1    D4    D7    D6    D5         (1101)
                 F1    F2    F3    F0    F5    F6    F7    F4      (1111)
              E0    E3    E2    E1    E4    E7    E6    E5         (1110)
                 A1    A2    A3    A0    A5    A6    A7    A4      (1010)
              B0    B3    B2    B1    B4    B7    B6    B5         (1011)
                 91    92    93    90    95    96    97    94      (1001)
              80    83    82    81    84    87    86    85         (1000)
```

Figure 29. 128 QAM case 2.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 30.

```
   01----02----03----00----05----06----07----04
10--|-13----12----11----14----17----16----15  |
 | 31    32    33    30    35    36    37  | 34
20  | 23    22    21    24    27    26    25  |
 | 61    62    63    60    65    66    67  | 64
70  | 73    72    71    74    77    76    75  |
 | 51    52    53    50    55    56    57  | 54
40  | 43    42    41    44    47    46    45  |
 | C1    C2    C3    C0    C5    C6    C7  | C4
D0  | D3    D2    D1    D4    D7    D6    D5  |
 | F1    F2    F3    F0    F5    F6    F7  | F4
E0  | E3    E2    E1    E4    E7    E6    E5  |
 | A1    A2    A3    A0    A5    A6    A7  | A4
B0  | B3    B2    B1    B4    B7    B6    B5  |
 | 91----92----93----90----95----96----97--|-94
80----83----82----81----84----87----86----85
```

Figure 30. subset constellation of 128 QAM case 2.

Case 3.3. Remove the third most protected bit in I. Figure 31 shows the region created in this case.

remove the third most protected bit of I

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0         Q
               0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    00    03    02    07    06    05    04      (0000)
              10    11    12    13    16    17    14    15         (0001)
                 31    30    33    32    37    36    35    34      (0011)
              20    21    22    23    26    27    24    25         (0010)
                 61    60    63    62    67    66    65    64      (0110)
              70    71    72    73    76    77    74    75         (0111)
                 51    50    53    52    57    56    55    54      (0101)
              40    41    42    43    46    47    44    45         (0100)
                 C1    C0    C3    C2    C7    C6    C5    C4      (1100)
              D0    D1    D2    D3    D6    D7    D4    D5         (1101)
                 F1    F0    F3    F2    F7    F6    F5    F4      (1111)
              E0    E1    E2    E3    E6    E7    E4    E5         (1110)
                 A1    A0    A3    A2    A7    A6    A5    A4      (1010)
              B0    B1    B2    B3    B6    B7    B4    B5         (1011)
                 91    90    93    92    97    96    95    94      (1001)
              80    81    82    83    86    87    84    85         (1000)
```

Figure 31. 128 QAM case 3.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 32.

```
   01----00----03----02----07----06----05----04
10--|-11----12----13----16----17----14----15   |
 | 31    30    33    32    37    36    35  | 34
20  | 21    22    23    26    27    24    25   |
 | 61    60    63    62    67    66    65  | 64
70  | 71    72    73    76    77    74    75   |
 | 51    50    53    52    57    56    55  | 54
40  | 41    42    43    46    47    44    45   |
 | C1    C0    C3    C2    C7    C6    C5  | C4
D0  | D1    D2    D3    D6    D7    D4    D5   |
 | F1    F0    F3    F2    F7    F6    F5  | F4
E0  | E1    E2    E3    E6    E7    E4    E5   |
 | A1    A0    A3    A2    A7    A6    A5  | A4
B0  | B1    B2    B3    B6    B7    B4    B5   |
 | 91----90----93----92----97----96----95--|-94
80----81----82----83----86----87----84----85
```

Figure 32. subset constellation of 128 QAM case 3.

Case 3.4. Remove the least protected bit in I. Figure 33 shows the region created in this case.

remove the least protected bit of I

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0        Q
               0} 0} 1} 1} 1} 1} 0} 0} 0} 0} 1} 1} 1} 1} 0} 0}

                 00    01    03    02    06    07    05    04     (0000)
              10    11    13    12    16    17    15    14        (0001)
                 30    31    33    32    36    37    35    34     (0011)
              20    21    23    22    26    27    25    24        (0010)
                 60    61    63    62    66    67    65    64     (0110)
              70    71    73    72    76    77    75    74        (0111)
                 50    51    53    52    56    57    55    54     (0101)
              40    41    43    42    46    47    45    44        (0100)
                 C0    C1    C3    C2    C6    C7    C5    C4     (1100)
              D0    D1    D3    D2    D6    D7    D5    D4        (1101)
                 F0    F1    F3    F2    F6    F7    F5    F4     (1111)
              E0    E1    E3    E2    E6    E7    E5    E4        (1110)
                 A0    A1    A3    A2    A6    A7    A5    A4     (1010)
              B0    B1    B3    B2    B6    B7    B5    B4        (1011)
                 90    91    93    92    96    97    95    94     (1001)
              80    81    83    82    86    87    85    84        (1000)
```

Figure 33. 128 QAM case 4.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 34.

```
   00----01----03----02----06----07----05----04
10--|-11----13----12----16----17----15----14  |
 | 30    31    33    32    36    37    35   | 34
20  | 21    23    22    26    27    25    24  |
 | 60    61    63    62    66    67    65   | 64
70  | 71    73    72    76    77    75    74  |
 | 50    51    53    52    56    57    55   | 54
40  | 41    43    42    46    47    45    44  |
 | C0    C1    C3    C2    C6    C7    C5   | C4
D0  | D1    D3    D2    D6    D7    D5    D4  |
 | F0    F1    F3    F2    F6    F7    F5   | F4
E0  | E1    E3    E2    E6    E7    E5    E4  |
 | A0    A1    A3    A2    A6    A7    A5   | A4
B0  | B1    B3    B2    B6    B7    B5    B4  |
 | 90----91----93----92----96----97----95--|-94
80----81----83----82----86----87----85----84
```

Figure 34. subset constellation of 128 QAM case 4.

Case 3.5. Remove the most protected bit in Q. Figure 35 shows the region created in this case.

remove the most protected bit of Q

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
               0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0         Q
              0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    0D    0E    0B    08     (000)
              10    13    16    15    1C    1F    1A    19        (001)
                 31    32    37    34    3D    3E    3B    38     (011)
              20    23    26    25    2C    2F    2A    29        (010)
                 61    62    67    64    6D    6E    6B    68     (110)
              70    73    76    75    7C    7F    7A    79        (111)
                 51    52    57    54    5D    5E    5B    58     (101)
              40    43    46    45    4C    4F    4A    49        (100)
                 41    42    47    44    4D    4E    4B    48     (100)
              50    53    56    55    5C    5F    5A    59        (101)
                 71    72    77    74    7D    7E    7B    78     (111)
              60    63    66    65    6C    6F    6A    69        (110)
                 21    22    27    24    2D    2E    2B    28     (010)
              30    33    36    35    3C    3F    3A    39        (011)
                 11    12    17    14    1D    1E    1B    18     (001)
              00    03    06    05    0C    0F    0A    09        (000)
```

Figure 35. 128 QAM case 5.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 36.

```
   01----02----07----04----0D----0E----0B----08
10--|-13----16----15----1C----1F----1A----19  |
 | 31    32    37    34    3D    3E    3B  | 38
20  | 23    26    25    2C    2F    2A    29  |
 | 61    62    67    64    6D    6E    6B  | 68
70  | 73    76    75    7C    7F    7A    79  |
 | 51    52    57    54    5D    5E    5B  | 58
40  | 43    46    45    4C    4F    4A    49  |
 | 41    42    47    44    4D    4E    4B  | 48
50  | 53    56    55    5C    5F    5A    59  |
 | 71    72    77    74    7D    7E    7B  | 78
60  | 63    66    65    6C    6F    6A    69  |
 | 21    22    27    24    2D    2E    2B  | 28
30  | 33    36    35    3C    3F    3A    39  |
 | 11----12----17----14----1D----1E----1B--|-18
00----03----06----05----0C----0F----0A----09
```

Figure 36. subset constellation of 128 QAM case 5.

Case 3.6. Remove second most protected bit in Q. Figure 37 shows the region created in this case.

remove the second most protected bit of Q

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
               0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0        Q
              0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    0D    0E    0B    08     (000)
              10    13    16    15    1C    1F    1A    19        (001)
                 31    32    37    34    3D    3E    3B    38     (011)
              20    23    26    25    2C    2F    2A    29        (010)
                 21    22    27    24    2D    2E    2B    28     (010)
              30    33    36    35    3C    3F    3A    39        (011)
                 11    12    17    14    1D    1E    1B    18     (001)
              00    03    06    05    0C    0F    0A    09        (000)
                 41    42    47    44    4D    4E    4B    48     (100)
              50    53    56    55    5C    5F    5A    59        (101)
                 71    72    77    74    7D    7E    7B    78     (111)
              60    63    66    65    6C    6F    6A    69        (110)
                 61    62    67    64    6D    6E    6B    68     (110)
              70    73    76    75    7C    7F    7A    79        (111)
                 51    52    57    54    5D    5E    5B    58     (101)
              40    43    46    45    4C    4F    4A    49        (100)
```

Figure 37. 128 QAM case 6.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 38.

```
   01----02----07----04----0D----0E----0B----08
10--|-13----16----15----1C----1F----1A----19  |
 | 31    32    37    34    3D    3E    3B  | 38
20  | 23    26    25    2C    2F    2A    29  |
 | 21    22    27    24    2D    2E    2B  | 28
30  | 33    36    35    3C    3F    3A    39  |
 | 11    12    17    14    1D    1E    1B  | 18
00  | 03    06    05    0C    0F    0A    09  |
 | 41    42    47    44    4D    4E    4B  | 48
50  | 53    56    55    5C    5F    5A    59  |
 | 71    72    77    74    7D    7E    7B  | 78
60  | 63    66    65    6C    6F    6A    69  |
 | 61    62    67    64    6D    6E    6B  | 68
70  | 73    76    75    7C    7F    7A    79  |
 | 51----52----57----54----5D----5E----5B--|-58
40----43----46----45----4C----4F----4A----49
```

Figure 38. subset constellation of 128 QAM case 6.

Case 3.7. Remove the third most protected bit in Q. Figure 39 shows the region created in this case.

remove the third most protected bit of Q

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
               0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0         Q
              0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    0D    0E    0B    08     (000)
              10    13    16    15    1C    1F    1A    19        (001)
                 11    12    17    14    1D    1E    1B    18     (001)
              00    03    06    05    0C    0F    0A    09        (000)
                 21    22    27    24    2D    2E    2B    28     (010)
              30    33    36    35    3C    3F    3A    39        (011)
                 31    32    37    34    3D    3E    3B    38     (011)
              20    23    26    25    2C    2F    2A    29        (010)
                 61    62    67    64    6D    6E    6B    68     (110)
              70    73    76    75    7C    7F    7A    79        (111)
                 71    72    77    74    7D    7E    7B    78     (111)
              60    63    66    65    6C    6F    6A    69        (110)
                 41    42    47    44    4D    4E    4B    48     (100)
              50    53    56    55    5C    5F    5A    59        (101)
                 51    52    57    54    5D    5E    5B    58     (101)
              40    43    46    45    4C    4F    4A    49        (100)
```

Figure 39. 128 QAM case 7.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 40.

```
   01----02----07----04----0D----0E----0B----08
10--|-13----16----15----1C----1F----1A----19  |
 | 11    12    17    14    1D    1E    1B  | 18
00  | 03    06    05    0C    0F    0A    09  |
 | 21    22    27    24    2D    2E    2B  | 28
30  | 33    36    35    3C    3F    3A    39  |
 | 31    32    37    34    3D    3E    3B  | 38
20  | 23    26    25    2C    2F    2A    29  |
 | 61    62    67    64    6D    6E    6B  | 68
70  | 73    76    75    7C    7F    7A    79  |
 | 71    72    77    74    7D    7E    7B  | 78
60  | 63    66    65    6C    6F    6A    69  |
 | 41    42    47    44    4D    4E    4B  | 48
50  | 53    56    55    5C    5F    5A    59  |
 | 51----52----57----54----5D----5E----5B--|-58
40----43----46----45----4C----4F----4A----49
```

Figure 40. subset constellation of 128 QAM case 7.

Case 3.8. Remove the least protected bit in Q. Figure 41 shows the region created in this case.

remove the least protected bit of Q

```
I-value {}I   {0 {0 {0 {0 {0 {0 {0 {0 {1 {1 {1 {1 {1 {1 {1 {1
Q-value ()     0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
               0  0  1  1  1  1  0  0  0  0  1  1  1  1  0  0        Q
              0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0} 0} 1} 1} 0}

                 01    02    07    04    0D    0E    0B    08    (000)
              00    03    06    05    0C    0F    0A    09       (000)
                 11    12    17    14    1D    1E    1B    18    (001)
              10    13    16    15    1C    1F    1A    19       (001)
                 31    32    37    34    3D    3E    3B    38    (011)
              30    33    36    35    3C    3F    3A    39       (011)
                 21    22    27    24    2D    2E    2B    28    (010)
              20    23    26    25    2C    2F    2A    29       (010)
                 61    62    67    64    6D    6E    6B    68    (110)
              60    63    66    65    6C    6F    6A    69       (110)
                 71    72    77    74    7D    7E    7B    78    (111)
              70    73    76    75    7C    7F    7A    79       (111)
                 51    52    57    54    5D    5E    5B    58    (101)
              50    53    56    55    5C    5F    5A    59       (101)
                 41    42    47    44    4D    4E    4B    48    (100)
              40    43    46    45    4C    4F    4A    49       (100)
```

Figure 41. 128 QAM case 8.

The resulting non-square constellation is the superposition of two square constellations, one of those had been shifted. This is shown in Figure 42.

```
   01----02----07----04----0D----0E----0B----08
00--|-03----06----05----0C----0F----0A----09  |
 | 11    12    17    14    1D    1E    1B  | 18
10  | 13    16    15    1C    1F    1A    19  |
 | 31    32    37    34    3D    3E    3B  | 38
30  | 33    36    35    3C    3F    3A    39  |
 | 21    22    27    24    2D    2E    2B  | 28
20  | 23    26    25    2C    2F    2A    29  |
 | 61    62    67    64    6D    6E    6B  | 68
60  | 63    66    65    6C    6F    6A    69  |
 | 71    72    77    74    7D    7E    7B  | 78
70  | 73    76    75    7C    7F    7A    79  |
 | 51    52    57    54    5D    5E    5B  | 58
50  | 53    56    55    5C    5F    5A    59  |
 | 41----42----47----44----4D----4E----4B--|-48
40----43----46----45----4C----4F----4A----49
```

Figure 42. subset constellation of 128 QAM case 8.

3.5. Higher order modulations

For the cases above 128 QAM the same rules applied.

4. Impact in the Computational Complexity

In order exploit the reduction of processing complexity of separable I and Q constellations together with the creation of normally non-separable constellations, the non-separable constellation is creating using constituent separable constellations.

For example, normally non-separable 8 points constellation can be created:

```
                (-1,+3)           (+3,+3)
        (-3,+1)          (+1,+1)
                (-1,-1)           (+3,-1)
        (-3,-3)          (+1,-3)
```

with the symbols assignments of:

```
                  001               010
          100               111
                  101               110
          000               011
```

and the point assignments of:

```
                 p31               p33
         p20               p22
                 p11               p13
         p00               p02
```

by combining the separable constellations:

constellation A defined as:

```
        (-3,+1)          (+1,+1)
        (-3,-3)          (+1,-3)
```

with the symbols assignments of:

```
          100               111
          000               011
```

and the point assignments of:

```
         p20               p22
         p00               p02
```

constellation B defined as:

```
        (-1,+3)           (+3,+3)
        (-1,-1)           (+3,-1)
```

with the symbols assignments of:

```
          001               010
          101               110
```

and the point assignments of:

```
         p31               p33
         p11               p13
```

Again extracting the value for the least significant bit as:

$$value = \frac{S1}{S0} = \frac{\sum_{bit=1} e^{(-n^* m_{ij})}}{\sum_{bit=0} e^{-(n^* m_{ij})}} \qquad ; ij = 31,11,22,02 \quad ; ij = 20,00,33,13, \tag{23}$$

$$= \frac{S1A + S1B}{S0A + S0B} \tag{24}$$

where:

$$S1A = \sum e^{(-n^* m_{ij})} \qquad ij = 22, 02 \tag{25}$$

$$S1B = \sum e^{(-n^* m_{ij})} \qquad ij = 31, 11 \tag{26}$$

$$S0A = \sum e^{(-n^* m_{ij})} \qquad ij = 20, 00 \tag{27}$$

$$S0B = \sum e^{(-n^* m_{ij})} \qquad ij = 33, 13 \tag{28}$$

Since constellations A and B have separable I and Q,

$$S1A \quad = \quad SyA \quad Sx1A \tag{29}$$

$$S1B \quad = \quad SyB \quad Sx1B \tag{30}$$

$$S0A \quad = \quad SyA \quad Sx0A \tag{31}$$

$$S0B \quad = \quad SyB \quad Sx0B \tag{32}$$

$$Sx1A = \sum e^{(-n^* mxj)} \qquad j = 2 \tag{33}$$

$$Sx0A = \sum e^{(-n^* mxj)} \qquad j = 0 \tag{34}$$

$$SyA = \sum e^{(-n^* myi)} \qquad i = 2, 0 \tag{35}$$

$$Sx1B = \sum e^{(-n^* mxj)} \qquad j = 1 \tag{36}$$

$$Sx0B = \sum e^{(-n^* mxj)} \qquad j = 3 \tag{37}$$

$$SyB = \sum e^{(-n^* myi)} \qquad i = 3, 1 \tag{38}$$

and the value extracted from the channel for the least significant bit would be:

$$value = \frac{S1A + S1B}{S0A + S0B} = \frac{SyA \;\; Sx1A + SyB \;\; Sx1B}{SyA \;\; Sx0A + SyB \;\; Sx0B} \tag{39}$$

Additional reduction of computation can be achieved by recognizing that SyA and SyB are identical for the second least significant bit.

As an example of the complexity reduction, consider a large QAM constellation, say 128 symbols, that was created from two constituent 64 bit constellations. The complexity for both full and reduced calculations are, for all bits:

128 QAM	# exp.	#adds	#mul	#div	TOTAL
Full	32	7*126	121	7	1,042
Reduced	32	7*14+2*14	14	14	186

N odd

$N=2^n$ QAM	# exp.	#adds	#mul	#div	TOTAL
Full	$2(2N)^{1/2}$	n(N-2)	N-n	n	$2(2N)^{1/2}+(n+1)N-2n$
Reduced	$2(2N)^{1/2}$	$2n^2+4n$	2n	2n	$2(2N)^{1/2}+2n^2+8n$

The increase complexity for this type of constellation can be shown to be of $O((N)^{1/2})$ where N is the number of constellation points.

5. Conclusion

We propose a way to use non-squared constellations that uses constituent independent I&Q constellations. We show the computational complexity advantages that the use of these constellations provide. This technique is applicable to any Receiver soft-Decision Decoding Technique, such as Turbo codes or LDPC codes.

6. Summary

We recommend that G.992.1.bis, G.992.2.bis and G.vdsl use non-square constellations base in this technique.

1. Agenda Item: G.992.1.bis issue 4.2 and G.992.2.bis issue 1.4. G.vdsl issue 11.17
2. Expectations: The committee accepts the technique described in this paper.

7. References

[1] NF-084. “Concatenated Trellis/Reed-Solomon Coding in DMT Systems”. Kschischang, Forney & Eyuboglu. Motorola, Nice France, 11-14 May 1998. http://ties.itu.int/u/tsg15/sg15/wp1/q4/98-05-Nice/NF-084.pdf .

[2] NF-083. “A new Approach to PAR Control in DMT Systems”. Kschischang, Narula & Eyuboglu. Motorola, Nice France, 11-14 May 1998. http://ties.itu.int/u/tsg15/sg15/wp1/q4/98-05-Nice/NF-083.pdf

[3] AB-093. “G.gen: Use of Parallel Concatenated Convolutional Codes PCCC (Turbo-Codes) for G.dmt and G.lite”. V. Demjanenko, J.A.Torres, Antwerp, Belgium, 3-7 August 1998. http://ties.itu.int/u/tsg15/sg15/wp1/q4/98-08-Antwerp/AB-093R1.doc .

[4] AB-120. “G.dmt: Text to include an optional Parallel Concatenated Convolutional Codes (PCCC) in the draft G.dmt Recommendation”, V. Demjanenko, J.A. Torres. Antwerp, Belgium, 3-7 August 1998. http://ties.itu.int/u/tsg15/sg15/wp1/q4/98-08-Antwerp/AB-120.doc .

[5] T1E1.4/98-300. “ Text to include an optional Concatenated Convolutional Cede in ANSI T1.413 Issue2”. J. A. Torres, V. Demjanenko. San Antonio, Texas, August 31- September

[6] T1E1.4/98-301. “ Inclusion of Concatenated Convolutional Cede in ANSI T1.413 Issue2”. J. A. Torres, V. Demjanenko. San Antonio, Texas, August 31- September

[7] D.376 “Text to include an optional Serial Concatenated Convolutional Codes in the next version of G.dmt”. J. A. Torres, V. Demjanenko. Geneva, Switzerland, October 12-23 1998.

[8] PO-071. “G.dmt: Inclusion of a Serial Concatenated Convolutional Code in the G.992.1.bis”. J. A. Torres, V. Demjanenko. Sunriver, Oregon 18-22 January 1999. http://ties.itu.int/u/tsg15/sg15/wp1/q4/99-01-Portland/PO-071.zip.

[9] NG-097. “G.gen: Considerations about the power penalty of Reed-Solomon Forward Error Correction in ADSL systems in regard to the use of inner encoders”. J. A. Torres, F Hirzel &

V. Demjanenko. Nuremberg, Germany 2-6 August 1999. http://ties.itu.int/u/tsg15/sg15/wp1/q4/99-08-Nurenberg/NG-097.zip.
[10] NT-120. "G.dmt: Inclusion of a Serial Concatenated Convolutional Code in the G.992.1.bis". F. Hirzel, J. A. Torres, V. Demjanenko. Nashville, Tennessee 1-5 November 1999. http://ties.itu.int/u/tsg15/sg15/wp1/q4/99-11-Nashville/NT-120.zip.
[11] BA-020R1. "G.gen: New proposal of Turbo Codes for ADSL modems". J. A. Torres, F. Hirzel & V. Demjanenko. Antwerp, Belgium 19-23 June 2000. http://ties.itu.int/u/tsg15/sg15/wp1/q4/00-06-Antwerp/BA-020R1.pdf .
[12] BA-108R1. "G.gen: Coding Ad Hoc Report". Ad Hoc group, Antwerp, Belgium 19-23 June 2000. http://ties.itu.int/u/tsg15/sg15/wp1/q4/00-06-Antwerp/BA-108R1.doc.
[13] HC-073. "G.gen: Results of the Requeriments requested in the Coding Ad Hoc Report (BA-108R1) for the proposed Turbo Codes for ADSL modems by VOCAL Technologies Ltd. in BA-020R1". J. A. Torres, F. Hirzel & V. Demjanenko. Huntsville, Canada 31 July- 04 August 20009. http://ties.itu.int/u/tsg15/sg15/wp1/q4/00-08-Huntsville/HC-073.pdf .
[14] "Turbo Coding". Chris Heegard, Stephen B. Wicker, Kluwer Academic Publishers, 1999.
[15] "Turbo Codes. Principles and Applications". Branka Vucetic & Jinhong Yuan. Kluver Academic Publishers, 2000.
[16] BM-087. "G.gen: Comparison of simulation results for different Coding Techniques (Uncoded, Reed-Solomon, Reed-Solomon plus Trellis and Reed-Solomon plus Parallel Concatenated Convolutional Codes) for G.992.1.bis and G.992.2.bis". J. A. Torres, Frederic Hirzel, Victor Demjanenko. Boston 10-14 May 1999. http://ties.itu.int/u/tsg15/sg15/wp1/q4/99-05-Boston/BM-087.doc .
[17] "Parallel Concatenated Trellis Coded Modulation". Benedetto, Divsalar, Montorsi & Pollara. *http://www331.jpl.nasa.gov/public/ ttcmICC96new.pdf* .
[18] "Soft-Output Decoding Algorithms in iterative Decoding of Turbo Codes". Benedetto, Divsalar, Montorsi & Pollara. TDA 42-124 February 1996. http://www331.jpl.nasa.gov .
[19] "Iterative decoding of Turbo Codes and other Concatenated Codes". Sorin Adrian Barbulescu. Ph. D. Thesis University of South Australia. 1996. http://www.itr.unisa.edu.au/~steven/thesis/sab.ps.gz.
[20] "DSP implementation of the Newly proposed CCSDS Telemetry Channel Coding Standard" 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000, pages 247-250. G. Montorsi, P.Coccia, A. Perotti, R, Garello, R. Maggiora, S. Benedetto, A. Serra, E. Vassallo and G.P. Calzolari.
[21] "Turbo-Codec-QPSK modem for Intelsat Digital Services". 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000, pages 487-490. Sorin Adrian Barbulescu, Rory Chang and Salim Yaghmour.
[22] "Full Turbo Receiver for coded Asynchronous DS-CDMA over multipath channel". 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000, pages 495-498. A. Roumy, I. Fijalkow and D. Pirez.
[23] "The turbo Code Standard for DVB-RCS" 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000. page 535-538. C. Douillard, M. Jezequel, C. Berrou, N. Berngarth, J. Tousch and N. Pham.
[24] "DVD-RCS Turbo Code on a Commercial OPB Satellite Payload: SKYPLEX". 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000. page 551-554. N. Berngarth, R. Novello, N. Pham, V. Piloni and J. Tousch.
[25] "Bandwidth efficient turbo coding for high speed mobile satellite communications". 1st International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 1997. page 551-554. S.A. Barbulescu, Wade Farrel, Paul Gray and Mark Rice.
[26] "Versatile Bandwidth-efficient Parallel and Serial Turbo-Trellis-coded Modulation". 2nd International Symposium on Turbo Codes & Related Topics- Brest, France September 4-7, 2000. page 201-208. Sergio Benedetto and G. Montorsi.

[27] "Proposal on using Multi-Tone Turbo Trellis Coded Modulation" T1E1.4 meeting in Napa, CA 2-4 October 2000. AT&T and Mitel. Hamid R. Sadjadpour, Rajan Sonalkar and Gary Jin.